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Statistical Mesomechanics of Solid, Linking Coupled Multiple Space and Time Scales

This review begins with the description of a new challenge in solid mechanics: multiphysics and multiscale coupling, and its current situations. By taking spallation as an example, it is illustrated that the fundamental difficulty in these multiscale nonequilibrium problems is due to the hierarchy and evolution of microstructures with various physics and rates at various length levels in solids. Then, some distinctive thoughts to pinpoint the obstacles and outcome are outlined. Section 3 highlights some paradigms of statistical averaging and new thoughts to deal with the problems involving multiple space and time scales, in particular the nonequilibrium damage evolution to macroscopic failure. In Sec. 4, several frameworks of mesomechanics linking multiple space and time scales, like dislocation theory, physical mesomechanics, Weibull theory, and stochastic theory, are briefly reviewed and the mechanisms underlying the trans-scale coupling are elucidated. Then we turn to the frameworks mainly concerning damage evolution in Sec. 5, namely, statistical microdamage mechanics and its trans-scale approximation. Based on various trans-scale frameworks, some possible mechanisms governing the trans-scale coupling are reviewed and compared in Sec. 6. Since the insight into the very catastrophic transition at failure is closely related to strong trans-scale coupling, some new concepts on nonequilibrium and strong interaction are discussed in Sec. 7. Finally, this review is concluded with a short summary and some suggestions. "This review article cites 130 references." [DOI: 10.1115/1.2048654]

1 Introduction

The study of phenomena with coupled multiple space and time scales is a need and an opportunity. This is especially true for the problems in solid mechanics with microstructures. In particular, the main concern of coupled multiple space and time scales in solid mechanics should be put on those related to breakdown properties. The fundamental difficulty in the problems is due to the hierarchy and evolution of microstructures with various physics and rates at various length levels in solids. In order to form such a trans-scale theoretical framework to link coupled multiple space and time scales, the mechanisms governing how the mesoscopic kinetics are in balance with macroscopic equations of mechanics should be clarified.

Some typical frameworks are reviewed in this paper. The emphasis is put on their representations, characteristic mesoscopic length scales, the mechanisms governing the transfer of mesoscale parameters to macrobehaviors and their trans-scale formulations. For trans-scale damage evolution to failure, statistical microdamage mechanics, which deals with several length and time scales, is reviewed. For an illustrative problem—spallation, the Deborah numbers, namely, the ratios of multiple time scales—appear to be the key factors governing the multiscale process. Furthermore, the cascade of damage evolution magnifies the effects of microstructures on failure and induces trans-scale sensitivity. For the sake of predicting evolution induced catastrophe, the concept of critical sensitivity seems to be promising in practice.

In one word, the new challenge in statistical mesomechanics of solids is to deal with the coupled physics with multiple time scales at multiple space scales, to understand the nonequilibrium evolution to catastrophe of engineering significance, to establish the corresponding coupled trans-scale formulations, and to clarify the mechanisms underlying the trans-scale coupling.

The main goal of the present review is to clarify why the study of phenomena with coupled multiple space and time scales is a need and an opportunity and what the challenges are in characterizing multiscale phenomena, then to give some clues to the physical understanding and formulation of these problems. This is especially important for the problems in solid mechanics with microstructures and related to breakdown properties, that is, the failure of solid materials.

This review begins with the description of a new challenge in solid mechanics: multiphysics and multiscale coupling, and its current situations. By taking spallation as an example, it is illustrated that the fundamental difficulty in these multiscale nonequilibrium problems is due to the hierarchy and evolution of microstructures with various physics and rates at various length levels in solids. Then, some distinctive thoughts to pinpoint the obstacles and outcome are outlined. Section 3 highlights some paradigms of statistical averaging and new thoughts to deal with the problems involving multiple space and time scales, in particular the nonequilibrium damage evolution to macroscopic failure. In Sec. 4, several frameworks of mesomechanics linking multiple space and time scales, like dislocation theory, physical mesomechanics, Weibull theory, and stochastic theory, are briefly reviewed and the mechanisms underlying the trans-scale coupling are elucidated. Then we turn to the frameworks mainly concerning damage evolution in Sec. 5, namely, statistical microdamage mechanics and its trans-scale approximation. Based on various trans-scale frameworks, some possible mechanisms governing the trans-scale cou-
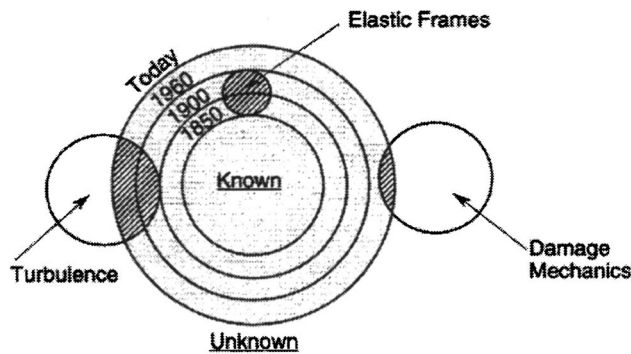
Fig. 1 Damage mechanics in the perspective of the expansion of human knowledge [3]

pling are reviewed and compared in Sec. 6. Since the insight into the very catastrophic transition at failure is closely related to strong trans-scale coupling, some new concepts on nonequilibrium and strong interaction are discussed in Sec. 7. Finally, this review is concluded with a short summary and some suggestions.

2 A New Challenge: Multiphysics and Multiscale Coupling

2.1 Background. "There is another sort of problems, i.e., strength and plasticity theory, for which even essential physical formulation is still not available for engineering applications," Tsien wrote about 40 years ago in his well-known book *Physical Mechanics* [1].

This statement also works from the physical point of view. "Despite the tremendous development of solid-state physics in this century, physicists have paid slight attention to how things break. In part, this neglect has occurred because the subject seems too hard. Cracks form at the atomic scale, extend to the macroscopic level, are irreversible, and travel far from equilibrium" [2].

Unfortunately, this problem still remains a compelling challenge until now. In a recent review article in *Applied Mechanics Reviews*, Bazant and Chen [3] indicated that theoreticians in mechanics of materials paid little attention to the question of scaling and size effect in failure. The reason was that all the theories that existed prior to the mechanics of distributed damage and quasi-brittle nonlinear fracture use a failure criterion in terms of stresses and strains exhibiting no size effect. Furthermore, they clearly drew the progressive expansion of human knowledge (Fig. 1). "Although much has been learned, it appears that damage mechanics is a formidable problem whose difficulty may be of the same dimension as turbulence," they wrote. In fact, the problem of turbulence "has occupied the best minds for over a century" and "it will take a long time to resolve completely" [3]. Then they identified a micromechanical basis of damage, statistical characteristics of size effect, etc., as necessary, and potentially profitable research topics for the immediate future.

What are the main causes for the long-lasting and still a future challenge? From engineering points of view, as noted by Becker et al. [4], in order to predict and prevent machines from failure, we should pay more attention to prognostics and health management. This might be a six-layer hierarchy of integrated predictive prognostics. For instance, for a vehicle, these are platform, system, subsystem, component, element and material. "Though mission demands are made at the top level, failure is initiated at the lowest level." In fact, the initial damage, like microcrack or microvoid, may come from the lower microstructual level in materials. But the evolution coupling these levels, like creep or interaction of microdamage, may lead to the eventual rupture of the whole platform. Perhaps, the recent tragedy of the Columbia accident may result from such a similar process with multiple length

and time scales (www.spaceref.com).

More specifically, Miller said in the opening address at the *7th International Congress of Fatigue*, that "as we enter the third millennium, i.e., beyond 2000AD, more catastrophic fatigue failures will occur as engineers push the limits of design even further due to demands for greater efficiency" and "both defect size and cyclic crack growth increments are measured on the submicro to atomic scales" [5].

Maybe owing to the similar feeling, Glimm and Sharp [6] proposed multiscale science as a challenge for the twenty-first century. They stated that multiscale science is the study of phenomena that couple distinct length and time scales and there is both a need and an opportunity to develop the methods of multiscale science.

2.2 Current Situation. Thus, what sort of methods have been developed for these and similar problems and what are the potential and limitations of these approaches should be examined at first.

As a brief overview of the approaches to solids with microstuctures, we should emphasize micromechanics and damage mechanics. Perhaps Eshelby's solution to inclusion [7], Mura's book [8], and Budiansky's review paper [9] are the representative of the early works dealing with microstructures in terms of micromechanics. The paradigm of the so-called micromechanics was to apply traditional continuum mechanics to the analysis of typical microstructural processes. In this way, people have understood a number of significant features of microstructures, such as eigenstrain in inclusion, etc. In this sense, the essence of micromechanics is a top-down approach to expand continuum mechanics to its minimum limitation.

Unlike micromechanics, damage mechanics deals with changes in microstructures as a continuum variable—damage, or say a new internal variable in thermodynamics, [10–13]. Continuum damage mechanics ignores either microscopic details or governing mechanisms at meso- and microlevels. Thus, it is not strange why Krajcinovic [13] asked whether the selection of damage parameters is an art or science. As a practical tool in engineering, it simply takes the average variations of mechanical properties in damaged materials as the measure of damage, for example, the degradation of elastic modulus. Therefore, continuum damage mechanics consists of two parts: evolution law and critical damage. However, both evolution law of damage and critical damage are phenomenological and empirical, without micro- or mesoscopic physical basis.

Although, the two approaches start from two ends of multiscales in solids and look very different, both of them are continuum mechanics in nature and have no business with any new statistical regulations of microstructures. Therefore, these cannot properly cope with the nonequilibrium evolution of distributed microdamage to failure.

Later, so-called mesomechanics was proposed, for instance, see [14]. Prompted by the rapid development of new materials and being a connection of microstructure and mechanics, "mesomechanics is a new research thrust to evolve noncontinuum mechanics." It will "undoubtedly bring forth drastic modifications in the existing mechanics theories and, probably, new mechanics concepts" [14].

On the other hand, multiscale problems have also been investigated and discussed beyond the mechanics community, especially in mathematics and computations [15,16], physics and chemistry [17], biology [18], as well as material sciences [19].

For the viewpoint of computational solid mechanics, most of these works contributed to the multiscale algorithms. Some papers provided informative views on the paradigms used in these methods. Roughly speaking, there are three types of modeling: atomistic modeling and dislocation dynamics, continuum modeling, and network or lattice simulations. For example, computational methods were developed at microscales in terms of discrete dislocation plasticity, nonlocal plasticity, and coupling of diffusion

and deformation [20]; at mesoscales by the continuum mesome-chanical finite element method [21]; and two-scale method for periodic mesoscopic configurations [22,23]. Combined continuum and atomistic modeling has been performed and discussed by the Caltech group and their co-workers [24]. Since failure appears to be a collective phenomenon and fluctuations cannot be neglected, several network and lattice models have been introduced [25–29]. The advantages of these simple models are their ability to repro-duce some essential features of failure. For instance, Hansen et al. [30], Schmittbuhl et al. [31], and Hansen and Schmittbuhl [32] proposed a fuse model to reveal the scaling properties of brittle fracture surfaces. It is shown that the large-scale universal rough-ness of brittle fracture surfaces is due to the fracture propagation being a damage coalescence process described by a stress-weighted percolation and the roughness exponent is in full accor-dance with the value in experiments.

The development of these multiscale studies promoted the es-tablishment of some new journals and a number of symposia, among which the following may have special interests. The jour-nal *Physical Mesomechanics* began in 1998 in Russia. "In the physical mesomechanics a solid under loading is considered as a multilevel self-organizing system where the plastic flow develops self-consistently as shear stability loss at micro-, meso-, and mac-roscale levels" [33]. The other newly founded journal is *SIAM Journal on Multiscale Modeling and Simulation* (MMS). As for the nature of multiscale problems, MMS manifested as follows. "Multiscale modeling is highly interdisciplinary, with develop-ments occurring independently across fields. A broad range of scientific and engineering problems involve multiple scales. Tra-ditional monoscale approaches have proven to be inadequate, even with the largest supercomputers, because of the range of scales and the prohibitively large number of variables involved. Thus, there is a growing need to develop systematic modeling and simulation approaches for multiscale problems" [34].

Similarly, a number of international symposia were held to ex-plore the perspectives and new paradigms to cope with multiscale problems. For example, MRS Symposium on Advances in Mate-rials Theory and Modeling—Bridging over Multiple-Length and Time Scales [19], the series of Mesomechanics [35,36], Interna-tional Symposium of Multiscaling in Mechanics [37], etc.

From the variety of publications and activities, one may ask what are the new exact requirements involved in the problems with multiple scales? In order to understand this, we turn to a specific problem to illustrate why the available approaches are not enough to deal with this kind of multiple scale problem and what the new challenge is.

2.3 Illustrative Example With Multiple Space and Time Scales: Spallation. On the spectrum of damage accumulation, there are two extreme time-dependent cases: creep, i.e., the rup-ture under long-lasting load from hours to years; and spallation, the rupture under transient loading like nano- to microseconds. Both depend on the magnitude and duration of the imposed load. Clearly, the usual material properties, such as material strength, cease to be constant in the two cases. The six-layer hierarchy of integrated predictive prognostics [4] mentioned previously may result from similar processes. Thus, damage evolution is a nice example to illustrate the effects of coupled multiple space and time scales on macroscopic behaviors of materials. Since creep is quite familiar in the mechanics community, a brief overview is given on spallation below.

From experimental observations, a quite universal criterion of time-integral for spallation has been used for a long time [38],

$$(\sigma/\sigma^* - 1)^\nu \Delta t = K \tag{1}$$

where σ and σ^* are stress and a stress threshold, respectively, Δt is the load duration, and ν and K are two parameters. This crite-rion indicates that the critical stress to spallation is no longer a material constant, but a variable depending on its loading dura-tion. Furthermore, since the power exponent ν in the criterion is usually neither 1 nor 2, the criterion implies neither momentum nor energy criteria macroscopically [39–41]. Then what is the mechanism underlying the time-dependent failure? Actually, this is a common difficulty in dealing with time-dependent and multi-scale failure in solids. Perhaps, one way out is to explore the evolution of distributed microdamage at mesoscopical scale.

Furthermore, a number of interesting phenomena relevant to spallation were reported by Russian researchers. For instance, anomalous increase of steel spall strength was reported and inter-preted in terms of tensile stress relaxation resulting from marten-sitic transformation [41]; spall strength measurements for quartz single crystals by taking account of failure wave formation [42]. Some systematical summary of Russian scientists' observations and analysis of spallation in solids under shock-wave loading can be referred to Kanel et al. [43].

Davison and Stevens [44,45] extended the concept of con-tinuum damage to spallation. Particularly, they noted the differ-ence between simple and compound damage accumulation. Simple damage is a process, whose rate is independent of existing damage, at the early stage of damage evolution. Later, damage accumulation would become dependent on the amount of damage already present, since that for a damaged sample further damage may develop more easily than before. At this stage, the rate of damage accumulation would become a function of applied stress as well as the damage present. This is compound damage accu-mulation. They accurately pointed out the significance of com-pound damage in failure. However, what is the physical essence of compound damage has remained open for a long time.

In the 1990s, Meyers [46] and Grady and Kipp [47] made com-prehensive and critical reviews on the study of spallation. On the one hand, they noted that "the current availability of high-speed computers and shock-wave propagation codes makes possible the development of continuum models of fracture and fragmentation to include in these codes." On the other hand, they stressed that "we still need quantitative/predictive models based on continuum measure of spalling and nucleation-and-growth of microcracks." "The continuum models based on the statistical nucleation and growth of brittle and ductile fracture appear to be an attractive approach, especially with a framework which provides some forms of a continuum cumulative-damage description of the evolving fracture state" [48]. Recently, owing to the accumulation of more data, models and engineering practices for various mate-rials, this challenging problem drew more attentions again [49,50]. As pointed by Clifton [49] that "dynamic failure by the growth and coalescence of grain-boundary microcracks involves the cooperative interactions of propagating cracks. Insight into such processes is required from the perspective of stochastic me-chanics and from computer simulations of the debonding of as-semblages of grains." In particular, how to take microstructural and microdamage evolution quantitatively into account becomes a focus of macroscopic failure problem. For instance, to explore the nonequilibrium statistical physics of distributed microdamage and its trans-scale coupling, which is different from continuum dam-age mechanics, appears to be a new opportunity.

Specifically, from the above statements one can draw the fol-lowing picture: spallation is a typical process with coupled mul-tiple space and time scales. At least, there are two length scales: the sample size at macroscopic level and the microdamage size at mesoscopic level. In addition, there is an atomic length scale. Furthermore, in order to analyze the phenomenon, one should introduce a representative volume—another length scale larger than microdamage size, but less than the characteristic scale of macroscopic inhomogeneity. In particular, for complete spallation, there is a newly emergent structure—the spalled region, which may have a submacroscopic length scale. On the other hand, there are, at least, three time scales: the stress wave loading duration macroscopically, the two mesoscopic characteristic times: nucle-ation time and growth time of microdamage. Also, there is an

atomic time scale, such as the atomic frequency. So, spallation does serve as an illustrative example with multiple space and time scales.

2.4 Distinctive Thoughts. As mentioned before, distributed microdamage as a kind of mesoscopic inhomogeneity often plays a critical role in failure analysis. Time-dependent spallation, fragmentation, creep [51] and deflagration to detonation transition in propellants and explosives [52] are all such notable examples of nonequilibrium evolution of mesoscopic entities. Scientifically speaking, for these problems with coupled multiple space and time scales, conventional approaches in solid mechanics are almost helpless.

For a long time, people have been aware that in mechanical measurements the scatter in strength is almost always greater than stiffness. This is because stiffness reflects the average behavior of microstructure, whereas strength is very sensitive to the microstructural details. Hansen et al. [53] has described the matter very clearly. "It has long been known that material properties may be strongly influenced by the presence of disorder. However, the sensitivity to the disorder is widely different. Transport properties, like conductance and elastic constants, are much less sensitive than breakdown properties such as material strength." Recently, Bochenek and Pyrz [54] also shared the similar idea, "there is a major difference between those systems, for which an average is sufficient, and those for which one has to construct many realizations of the microstructure. Averaging is often done by an effective medium approach and is suitable for predicting properties such as stiffness or conductivity. However, fracture or electrical breakdown will depend on specific details in the microstructure and usually averaging is not acceptable." As a matter of fact, the underlying essence of the difference is that the breakdown behaviors, like failure and strength, is governed by the trans-scale cascade of microdamage evolution, whereas the conventional linear transport behavior is something near the equilibrium state. Therefore, it is obvious that a coupled trans-scale framework is a necessity for the physical understanding of breakdown behaviors. Or, we should say that the conventional mechanical paradigm of solving the three field equations of continuum, momentum, and energy together with constitutive relation becomes inadequate for the coupled trans-scale problems. Therefore, we have to explore some new paradigm to combine the trans-scaled processes.

However, in the course, we encounter two major obstacles. One is how to properly close the trans-scale coupling, because this kind of trans-scale formulation usually leads to an endless hierarchy from macro- to microscales, like the Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy [55]. The other obstacle is that the averaging usually used in continuum mechanics and most available multiscale modeling almost always obscures the effects of some mesoscopic details, which may be magnified and become critical to macroscopic failure. These, we suppose, are the real challenges we are facing.

More broadly speaking, McDowell pointed out in the preface of the ASTM STP on applications of continuum damage mechanics to fatigue and fracture that these occur "at multiple length scales with coupling between these scales" and "rigorous treatment of nonuniformly distributed defects requires tools not yet fully developed in continuum damage mechanics." This strongly appeals to irreversible statistical thermodynamics [56].

Now, we should extract some guiding thoughts to these new requirements in the problems governed by coupled multiple length and time scales. That is to say, what are the key points in the would-be paradigms to deal with material failure sensitive to the details at lower levels? The following are some of the issues on which recent symposia and workshops focused.

Though the existence of mesoscale inhomogeneities and stress fluctuations have certainly been recognized by experimentalists and theoretical analysts, the issue of heterogeneous and nonequilibrium shock front dynamics on the mesoscale has largely been
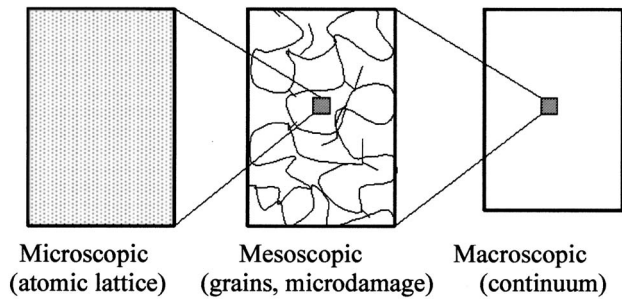

Fig. 2 Schematic illustration of the microscopic, mesoscopic, and macroscopic length scales in a polycrystal [59]

ignored, in spite of the fact that these must strongly influence the phenomena such as fracture and phase transitions. Thus, the following specific questions were posed in the workshop on *Shock Dynamics and Non-Equilibrium Mesoscopic Fluctuations in Solids* [57]: (1) What experimental data are available and what are their implications? (2) Are there new mesoscale theories for shock dynamics? (3) How do the theories affect the existing fracture and phase transition paradigms? (4) What kinds of new computational and materials models are needed?

In another workshop on *Multiscaling in Mechanics*, Sih wrote, "while multiscaling in mechanics is active, growing and developing new ideas, it should not be taken as the approach to solve the problem. It only reveals the complex entanglement of scale interactions that seemed to have no simple underlying form" [58]. Furthermore, he posed some provocative questions. (1) Is there a one to one correspondence between the material structure entities and macroscopic properties? (2) Is scaling in size and time a valid concept to pursue for understanding material behavior? (3) Is it mandatory to include the effect of imperfections if the bulk keeps on reducing its size?

Thus, to summarize the new challenge in statistical mesomechanics of solids, we will emphasize the trans-scale physics and corresponding multi-scale coupled formulations, which can reveal and represent the nonequilibrium damage evolution to failure of engineering significance.

3 Paradigms of Statistical Averaging and Coupling

Actually, looking at the space and time scales in solids with microstructures, principally, we are facing problems across three levels: the microscopic, mesoscopic, and macroscopic levels (Fig. 2) [59]. Strictly speaking, the term of "microscopic" should be attributed to the processes at atomic level, such as dislocation, point defect, etc. In this review, we still follow this understanding. On the other hand, a lot of phenomena at mesoscopic level have been coined by prefix micro-, but actually occur at mesoscopic level, such as microcrack, microvoid, microdamage as well as microstructure (grain and phase structure). In order to facilitate communicating with the available references, in this review we retain these terms, but treat them as mesoscopic ones.

In this section, we first illustrate the importance of various physics on various levels. Then, we turn to the paradigm of statistical averaging. Finally, we discuss the specific challenges in the trans-scale paradigms to deal with nonequilibrium damage evolution to macroscopic failure.

3.1 Various Physics at Various Levels. There are many illustrative examples to demonstrate the different physics at different levels [60]. To link two different level leads to statistical approaches. In history, the search for the essence of thermal phenomena led to thermodynamics. As continuum mechanics, thermodynamics, though correct and reliable, treats materials as continuum and then cannot provide the specific knowledge of a material. For the understanding of gas behavior, kinetic theory of

molecules was developed, particularly by Clausius, Maxwell, and Boltzmann. They found the relationship between the equation of state of gas and the statistical behavior of a great number of gas molecules, especially the velocity distribution of gas molecules. Based on these, Gibbs finished the general theory of ensemble statistics [61].

Statistical physics is an approach, different from classical mechanics, to study the properties of macroscopic bodies made up of a great number of particles. For a system consisting of many particles, even though one could find the general solution of the corresponding equations governing the particles, "it would be quite impossible to substitute in this the initial values of the particles, if only because of the time and paper it would consume" [62]. More than this, since macroscopic quantities are the statistical averages in nature, the adoption of a statistical approach is an objective necessity [61,63].

Recently, Kadanoff [64] pointed out that in order to examine fascinating properties of condensed matter systems, such as fracture of materials, and earthquake or avalanche dynamics, there has been an exposition and reorganization in the intensity and variety of research what one might call physical dynamics. As science turns to more and more complex systems, it might be that a statistical approach will become a crucial input to the next generation of scientific issues. Also, it should be partially grounded in experiment and partially in a deep analysis of the consequences of the laws of mechanics [64].

On the other hand, one may ask if statistical mechanics is a consequence of classical mechanics. Actually, "there appear new and distinctive regularities" in the system consisting of many particles. "These so-called statistical laws, which arise as a result of there being a large number of particles in the body, can never be explained in purely mechanical terms," or say "the importance of statistics lies in the fact that in nature we are dealing all the time with macroscopic bodies whose behaviour cannot be described by purely mechanical methods and which do, in fact, obey statistical laws" as Landau and Lifshitz wrote [62]. Why? This is because, under a certain macroscopic conditions the possibilities of microscopic states are numerous and can not be uniquely determined by the macroscopic condition. The contribution of all possible microscopic states to the statistical average is the new statistical law [61].

Moreover, the fundamental difficulty in nonequilibrium problems with multiple space and time scales in solid mechanics is due to the hierarchy of microstructures with various physics at various levels in solids. To understand this difficulty, first let us recall how to treat a perfect gas and a simple solid in traditional statistical physics.

Perfect gas (treated as system consisting of nearly free particles) and simple solid (treated as system consisting of linear oscillators) under equilibrium are the simplest cases. Taking perfect gas as an example, there are only two levels, molecules and continuum, and there is no interaction between molecules except for the moment of collision. Therefore, in order to link the two levels, we need only two constants: Avogadro constant $N_A = 6.02 \times 10^{23}$ mol/mole (the total of molecules) and Boltzmann constant $k = 1.38 \times 10^{-23}$ J/K (a constant introduced by Boltzmann statistics). Then, the equation of state of perfect gas can be derived by the physical essence of macroscopic pressure p proportional to the total kinetic energy of translational motion of all molecules and Boltzmann statistics [61,62]

$$pv_a = N_A k\theta = R\theta \tag{2}$$

where θ is temperature, ν_a is specific volume, $R = N_A k$ is molar gas constant, a previously macroscopic empirical constant, but with a clear microscopic physical basis (see Fig. 3).

Based on a similar idea but with a bit more calculations, the Grüneisen equation of state was established for solids [65]. In this approximate formulation, the crystalline solids are treated as a simple dynamic system, consisting only of a number of linear
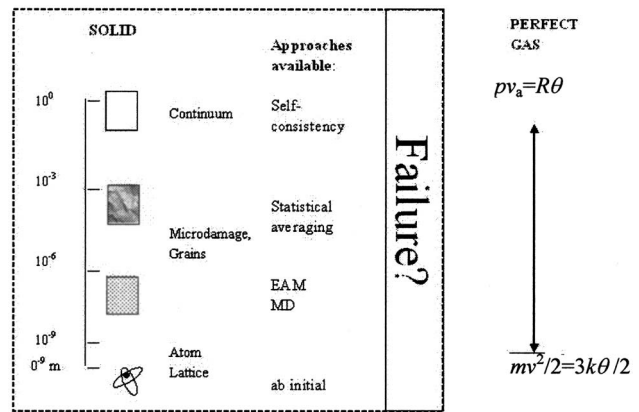

Fig. 3 Microscopic and macroscopic scales in traditional statistical physics

oscillators. Also, the only stress is pressure. In this way, Grüneisen equation of state of solids has a form similar to perfect gas and characterized by Grüneisen coefficient Γ, like the gas constant

$$[p - p_k(V)]V = \Gamma[E - E_k(V)] \tag{3}$$

where p_k and E_k are "cold" pressure and energy, respectively.

These two examples demonstrate two significant points. One is that the equation of state is by no means a system of motion equations of particles, but a new statistical law. The other is the reason why this trans-scale formulation is so simple lies in two aspects of these two systems: equilibrium and nearly no interaction of particles. Therefore, the principle of equal probability is valid and the probability distribution function is simple.

Looking back to the processes at mesoscopic levels of solids like damage evolution, we are not so lucky, Fig. 3. The problems at these levels are usually very dirty owing to nonequilibrium and strong interaction. One has to understand how the physical essence at lower levels is strongly coupled and transferred to a higher level during the course of nonequilibrium processes and how this influence on the higher level is formulated.

3.2 Paradigm of Statistical Averaging. Now, let us examine the main difficulties involved in the statistical paradigms linking multiple space and time scales in solids.

The fundamental principle of statistical physics is that macroscopic quantities are the statistical averages of their corresponding microscopic quantities [61]. However, when we examine a macroscopic quantity, such as density, we should examine a volume macroscopically small but microscopically large. If the volume is too large, then the macroscopic inhomogeneity cannot be characterized in terms of them; if the volume is too small, it presents strong statistical fluctuations. Also, the measurement of macroscopic quantities should be performed in a process macroscopically short but microscopically long, otherwise one cannot examine the macroscopically temporal variation of the field and cannot confirm the average resulting from a large number of particles. Hence, only when these assumptions are effective, the macroscopic quantities are the averages of all possible microscopic states under a given macroscopic conditions [61,63]. Still take gas as an example. Even in a cube of 1 mm³ there are still 2×10^{16} molecules and a molecular will collide with others 10^{10} times per second in the condition of standard atmosphere and zero degree centigrade. All these guarantee that the representative volume can be characterized by statistical averaging variables with small fluctuations.

Since statistical mechanics starts from equilibrium, it begs many interesting questions, as noted by Kadanoff [64]. "How long need we wait to attain equilibrium? Why do we say that all systems reach the same equilibrium independent of their dynamics?

Is this independence really true? How much can we say about the relaxation to equilibrium? How general is the mode of equilibrium? To begin to treat these questions one must have a dynamical perspective."

All these fundamental aspects in statistical mechanics become much more crucial in solids. For instance, for a representative volume element of 1 mm^3 containing grains of 1 μm in length, there are 10^9 grains. In addition, the frequency of an atom as an oscillator in solids, like the Debye frequency, is about 10^{12-13} Hz, but the characteristic time for microdamage growth is about 10^{-6} s. So, how to develop proper methods based on statistical mechanics to determine the size of the representative volume element is crucially important [66]. The dilemma is that the size of a representative, volume element should be large enough mesoscopically to be statistically representative, but small enough macroscopically to be a constitutive element in the macroscopic field. In "pure" deformation problems, i.e., only atomic events in crystallines are involved, a representative volume element of 1 mm^3 in usual time scales, like seconds, does satisfy the assumptions. Therefore, these problems can be solved by integrating continuum mechanics equations and a constitutive relation. In these cases, constitutive relation, though resulting from atomic processes, can be an independent formulation. However, for the phenomena like failure and breakdown, the correlation length may become very large and stress fluctuations become very significant. Also, the macroscopic imposed time scales may become comparable to the mesoscopic intrinsic ones, like the microdamage growth rate. From various numerical simulations it is found that these behaviors of such a system in which microstructural evolution-induced catastrophe occurs is governed by its mesoscopic configuration and stress fluctuations, not just their macroscopic mean values, hence show sample specificity [67,68]. Aidun et al. [66] suggested that the correlation length could be taken to define the linear dimension of an RV because it marks the crossover between small length scales, on which different regions of the system behave differently, and the large length scales, on which all regions of the system are comparable. Moreover, for the nonequilibrium processes of microdamage evolution, it is not certain whether the principle of equal probability can still work or not. Hence, thus far, how to form a general trans-scale formulation based on the ideas remains open yet.

3.3 Some Thoughts on Trans-Scale Coupling. Carefully examining some available approaches to multiscale problems, one may find that they are usually based on two fundamental assumptions. One is the self-similarity on various scales, and the other is the so-called adiabatic approximation, namely, the effects of smaller scales on behavior at a larger scale can be formulated by the averages on small scales. These approaches are very helpful for the properties, such as stiffness in engineering practices, but may not be suitable for damage evolution to failure.

Recently, He et al. and Wang et al. clarified this issue more clearly [69,70]. They stated that the essence of multiscale phenomena results from diversity and coupling on various length and time scales. There were two approaches developed in dynamics to deal with multiscale problems last century. For the problems with physical similarity on various scales, looking for similarity solutions is an effective approach. On the other hand, for the problems with weak coupling between two levels, perturbation methods with small parameters are very helpful. However, the multiscale problems we are facing now are different from those and the similarity solutions and perturbation methods are no longer suitable.

For the problems concerning evolution-induced catastrophe with multiscales, there are two fundamental difficulties. One is that there are different physics on different scales; therefore, self-similarity no longer works. The second is catastrophe is essentially sample specific; that is, the behavior of catastrophe on large scale will become sensitive to some details of microstructures on small scales so the adiabatic approximation does not work [67].

Furthermore, the damage and failure of solids under external loading are usually far from equilibrium state. Hence, equilibrium theories cannot be used to describe the phenomena properly. There are no simple direct connections between atomic and macroscopic features in the case of nonequilibrium process. A noticeable feature is the richness of structures and processes on mesoscopic length scales, such as grains, microvoids, microcracks, and their collective effects, which play a significant role in the phenomena.

Moreover, there are various mesoscopic kinetics with various time scales. Thus, not only length scales but also various time scales play significant roles in damage and failure [71].

Then what can be the key to the would-be paradigms in the coupled multiscale problems? Barenblatt [72] constructively proposed that, to determine the governing influence of the variations of the material microstructure on the macroscopic behavior of bodies, the macroscopic equations of mechanics and the kinetic equations of microstructual transformations should form a unified set that should be solved simultaneously. Additionally, he noted the significance of the Deborah number,

$$\text{De} = \frac{\tau(\text{relaxation time scale})}{\tau(\text{imposed time scale})} \tag{4}$$

The Deborah number implies the ratio of the characteristic relaxation time of a mesoscopic process to corresponding imposed macroscopic time scale in the problems. This idea may become crucially important in trans-scale formulations because the rate processes at the two levels can compete with each other by their characteristic time scales, as shown by Deborah number, though the ratio of mesoscopic and macroscopic length scales $R = l/L$ are very small. These ideas proposed by Barenblatt are very important in the problems with coupled multiple scales. Of course, there might be some mechanisms other than the competition of different rate processes. But, anyway, these ideas remind us of the importance of trans-scaled coupling mechanisms.

Therefore, some certain distinctive statistical approach should be explored on this track. The statistical mesomechanics linking multiple space and time scales is the main concern of this review.

4 Various Frameworks of Mesomechanics of Solids

As discussed in the previous section, the microstructures in solids at micro- and mesoscales are various and complicated. Unlike the kinetic theory of gas and the theory of linear oscillators of solids, the entity and representation in the statistical theory of mesomechanics of solids can vary depending on various problems and the various insights of researchers. So, since last century, a number of different frameworks have been developed. We will briefly review some typical frameworks, but put emphasis on those related to breakdown properties rather than transport ones.

4.1 Dislocation Theory. Probably, the dislocation theory is an early and far-reaching one. In order to answer the question why the force to break the crystal lattice is much higher than the force to cause plastic deformation, Orowen [73], Taylor [74], and Poliani [75] independently proposed the dislocation theory in 1934. Their answer is that plastic deformation is due to the motion of a number of dislocations. Clearly, the dislocation theory is not directly related to the breakdown behavior of solids, but it is very enlightening in linking multiple scales.

Although the mechanisms governing coupling effects of mesoscopic sizes on macroscopic behaviors can vary from case to case, the straightforward way for mesoscopic elements to affect macroscopic behavior is the sum of mesoscopic elements. Plasticity resulting from dislocation motion, characterized by a microscopic length scale, i.e., the dislocation Burgers vector b, is a most impressive example. According to Orowen [73], the continuum shear strain rate $\dot{\gamma}$ is proportional to the production of mobile dislocation density ρ_d, dislocation velocity v_d as well as the Burgers vector **b**,

$$\dot{\gamma} \approx \mathbf{b} \rho_d v_d \qquad (5)$$

In this expression, the shear strain rate $\dot{\gamma}$ is a macroscopic quantity, whereas the Burgers vector $\mathbf{b}$ and dislocation velocity v are both microscopic quantities. The characteristic microscopic length scale, namely, the Burgers vector, is coupled to the macroscopic shear strain rate by the total of dislocations in a macroscopic element, i.e., the dislocation density ρ_d. Similarly, Taylor's dislocation strengthening relation gives the relation between macroscopic shear flow stress and microscopic dislocation [74],

$$\tau \propto \mu \mathbf{b} \sqrt{\rho_T} \qquad (6)$$

where τ is shear flow stress, μ is shear modulus, $\mathbf{b}$ is Burgers vector, and ρ_T is total dislocation density, respectively. Hence, although the mechanism of the total of microscopic elements to enhance the microscopic length scale effect on macroscopic behavior is very simple and even trivial, the concept is very helpful in various applications.

Later, Kroener established a continuum theory of dislocations to describe macroscopically mechanical state of single and polycrystalline solids. It is a very general field theory to understand the phenomena of plasticity, residual stress, and lattice curvature [76,77]. Recently, in the light of Taylor's relation and Kroener's work, a mechanism-based strain gradient (MSG) plasticity theory was developed by Nix and Gao [78]. In addition, Dai et al. [79] applied Taylor's relation to interpret the particle strengthening effect in metal matrix composites.

Another new development on this track to bridge the atomic structures with macroscopic behavior in crystalline materials is the many-body force field (FF) calculation, developed by the group at Caltech and its coauthors recently [24]. They thought that the approach is a kind of "divide and conquer" paradigm. There are three steps. The first is to model the controlling unit process at microscopic scale. The second is to quantify the synergetics and dynamics of these mechanisms. Finally, the macroscopic driving force is correlated to macroscopic response. Clearly, the basis of this approach is to pinpoint proper unit processes. In their study of various aspects of plastic deformation, they focus on the processes related to dislocation mobility, interaction, and evolution, such as kinks, forest dislocation, dislocation multiplication, and annihilation. By making use of the framework, they successfully calculated the mechanical response of high-purity Ta single crystal [24].

For dislocations on the atomic scale, one could resort to quantum mechanics. However, for tangled dislocations and mesoscopic defects and damage, the descriptions in present-day theories are hardly complete. Large-scale computer modeling can provide insight into the behavior of solids as data are averaged at different scale levels; however, the conclusions are case specific [80].

4.2 Physical Mesomechanics. Physical mesomechanics, proposed by Panin and his co-workers, is another unified framework. In their theory, two mesoscopic levels are adopted. At level I, vortex plastic flow is based on dislocations and characterized by the scheme "shear+rotation;" whereas at level II new defects, such as banded mesostructures, appear irrespective of crystallographic orientations [33,81]. So, a solid in this framework is regarded as an ensemble of mesovolumes deformed by shear and rotation. In addition, continuum mechanics equations and dynamic gage theory of defects are used to describe the motion of mesovolume. They applied the framework to the study of the development of fatigue fracture under cyclic bending [81].

Actually, gage theory of defects can be traced back to Golebiewska-Lasota [82] and Golebiewska-Lasota and Edelen [83]. After comparing the defect field and the electromagnetic field, Golebiewska-Lasota applied gage theory to a continuum with dislocations or defects. Then precluding the use of the analogy with electrodynamics, from a gage-transformation point of view Golebiewska-Lasota and Edelen examined both dislocations

and disclinations, especially the underlying gage transformation structure and applied gage invariance to them. Generally speaking, the object of gage theory is a Lagrangian system with some symmetry. A continuum without defects has some symmetry, but the appearance of defects breaks it. For instance, the appearance of dislocations and disclinations in a solid changes its translational and rotational symmetry respectively [84]. So, the defect field is a gage field resulting from the defect-induced symmetry breaking of a basic continuum field, such as an elastic field.

In this sense, this framework is a development of dislocation and defect theories. Its elegant mathematical form is very impressive. However, its physical significance is not as straightforward as dislocation theory. Therefore, various gage theories of defects were developed by different authors [85]. Noticeably, unlike the Burgers vector $\mathbf{b}$ in physical dislocation theory, there is no longer a characteristic length scale of defects in the theory. This may limit the application of the theory to concrete engineering practice.

4.3 Weibull Theory. Different from dislocation theory, where dislocation is the entity of representation and plastic deformation is the question to answer, in Weibull theory the random strength of mesoscopic elements is taken to be the representation. In addition, in this formulation there is not any explicit or physical mesoscopic length scale at all, like the Burgers vector $\mathbf{b}$ in dislocation. According to Weibull [86], a macroscopic sample consists of N mesoscopic elements in series, which obey a distribution function $P(\sigma_c)$ of mesoscopic strength σ_c and may fail independently. Hence, the failure probability of the macroscopic sample is

$$F(\sigma) = 1 - [1 - P(\sigma)]^N \qquad (7)$$

where σ is the stress acting on mesoscopic elements. It is worth noting that the total of mesoscopic elements N is actually taken to be the ratio of macro- and mesoscopic length scales. Suppose that the distribution function $P(\sigma_c)$ follows Weibull distribution

$$P(\sigma_c) = 1 - \exp\left[-\left(\frac{\sigma_c}{\eta}\right)^\beta\right] \qquad (8)$$

where the characteristic strength η and the shape factor β (also Weibull modulus) are two trans-scale parameters. Then, the failure probability of the macroscopic sample becomes

$$F(\sigma) = 1 - \exp\left[-N\left(\frac{\sigma}{\eta}\right)^\beta\right] \qquad (9)$$

Accordingly, the expectation of the failure probability $F(\sigma)$ is taken to be the average macroscopic strength,

$$\sigma_f = \int_0^\infty \sigma \frac{dF}{d\sigma} d\sigma = \eta N^{-(1/\beta)} \Gamma\left(1 + \frac{1}{\beta}\right) \qquad (10)$$

Later, Coleman [87] indicated that fiber does follow the Weibull distribution. Then, it is usually deduced that there is a power law of scaling, Eq. (10), in Weibull theory. As noted before, N implies the size ratio between the macroscopic sample and its mesoscopic element. The latter is assumed to be a unit and without any fixed and characteristic physical value. Thus, Eq. (10) demonstrates that Weibull theory does not include any characteristic size and is irrelevant to any mesoscale material length scale [3].

The concept of Weibull theory has also been widely and successfully applied to bundle of fibers, fiber-reinforced composites, etc. [88–90], for example Daniel [88] showed that the strength of fiber bundle follows normal distribution, provided the number of fibers tend to infinity. Different from the sample consisting of mesoscopic elements in series, when a sample consists of N mesoscopic elements in parallel, for instance, parallel loose bundle [91], damage D can be expressed by

$$D = \int_0^{\sigma_s} \frac{dP}{d\sigma_c} d\sigma_c = 1 - \exp\left[-\left(\frac{\sigma_s}{\eta}\right)^{\beta} \right] \quad (11)$$

where σ_s is the true stress acting on mesoscopic elements. Then, the maximum nominal stress for a macroscopic sample, i.e., the macroscopic strength σ_f, will be

$$\sigma_f = \sigma_{\max} = \eta(e\beta)^{-(1/\beta)} \quad (12)$$

Although the macroscopic strength depends on the two trans-scale parameters β and η, Eq. (12) is independent of any mesoscopic length scales and the size ratio N.

Clearly, for the cases with mesoscopic elements in either series or parallel, Weibull statistical theory implies no characteristic length scales, as reviewed by Bazant and Chen [3]. Therefore, principally it may not be suitable to be applied to the materials and structures with microdamage, microcracks, or microvoids, provided these mesoscopic structures do not change with the macroscopic size of a sample.

4.4 Stochastic Theory. Real media are usually highly heterogeneous. However, in classical continuum mechanics, the collective effects of heterogeneity are usually packed into macroscopic constitutive relations by averaging. At most, some internal variables and their macroscopic empirical evolution laws are introduced as a complement to the constitutive relations. Obviously, this is insufficient to capture the complexity of macroscopic behaviors of heterogeneous media. In these situations, a comprehensive theoretical framework is needed to describe the formation and evolution of microstructures at appropriate length and time scales and to establish their connection with the corresponding macroscopic properties.

Glimm and Sharp and co-workers [6] thought that processes at micro- and mesolevels are stochastic and then only some kind of averages of microscopic physics can be observed macroscopically. Therefore, they proposed a general nonlinear framework of multiple scale problems,

$$\frac{\partial U}{\partial t} + \nabla F(U) = 0 \quad (13)$$

where U is a continuous but stochastic variable. After introducing the average of an ensemble, one can compare multiscale calculations with experimental observations.

In order to understand damage accumulation, especially creep, Barenblatt [72] proposed a stochastic description of damage field evolution. He also presumed that mesoscopic damage distribution and corresponding stress are stochastic. A phenomenological formulation of the kinetics of damage accumulation was assumed as

$$\frac{\partial \hat{D}}{\partial t} = \frac{q_b(\hat{D}, \sigma_s, \theta)}{\tau_r} \quad (14)$$

where $\hat{D}$ is local damage on a cross section, σ_s true stress, θ temperature, τ_r the relaxation time of damage accumulation, q_b a dimensionless function of $\hat{D}$, σ_s, θ, and q_b could be assumed to be of Arrhenius-type kinetics. After introducing the average over the cross section, for the average damage D, he derived

$$\frac{\partial D}{\partial t} = \frac{1}{\tau_r} \int_{-\infty}^{\infty} q_b(\hat{D}, \sigma_s(\xi), \theta) \rho_w(x - \xi) d\xi \quad (15)$$

where ρ_w is a symmetric weight function, which indicates the relative amount of elements (at ξ) affecting damage at x. After defining a microstructural length scale λ by

$$\lambda = \left\{ \int_{-\infty}^{\infty} (x - \xi)^2 \rho_w(x - \xi) dx / 2 \right\}^{1/2} \quad (16)$$

and taking the first two terms of the Taylor series of the function q_b, Barenblatt obtained the following nonlinear reaction-diffusion-type equation of damage evolution [72]:

$$\frac{\partial D}{\partial t} = \frac{q_b}{\tau_r} + \frac{\partial}{\partial x}\left[\left(\frac{\lambda^2 \sigma_s}{\tau_r} \frac{\partial q_b}{\partial \sigma_s} \right) \frac{1}{(1-D)} \frac{\partial D}{\partial x} \right] \quad (17)$$

This leads to a stress-influenced damage diffusion. In this way, the diffusion is dependent on the combination of stress, damage, relaxation time, and the length scale λ. This equation consists of continuum damage and its mesoscopic kinetics, as indicated by function q_b and parameters τ_r and λ.

The representations of dislocation and its extensions clearly link the microstructure of solids to plastic deformation on a macroscopic level very well, but not directly related to rupture of solids. Although Weibull theory concerns the problem of strength of solids and provides a certain size effect, it does not link a real micro- or mesoscopic characteristic length scale to macroscopic failure. As for the stochastic theory, like that proposed by Barenblatt, continuum damage was adopted as the representation and a mesoscopic length scale λ was introduced, but the length scale λ needs a realistic physical explanation in materials. Thus, perhaps from a mechanical point of view, we had better turn to the real entity of microdamage at the mesoscopic level in solids and to establish a more realistic formulation.

As we stated at the very beginning of this review, the main concern of statistical mesomechanics is the breakdown properties of solids resulting from microstructures with characteristic micro- or meso-length scales. In Sec. 5, we will turn to the representation of microdamage number density and continuum damage based on microdamage number density.

5 Statistical Microdamage Mechanics: Microdamage Number Density and Continuum Damage Based on Microdamage Number Density

If there is a crack in a solid, then fracture mechanics successfully characterizes the failure of the solid. However, for most heterogeneous materials, such as alloys, ceramics, composites, rocks, etc., there might be distributed microcracks or microvoids rather than a single macroscopical crack. Thus, instead of fracture mechanics, new theoretical formulations are needed.

For a piece of solid containing preexisting inhomogeneities, defects, or flaws, what kind of successive changes will appear and how will rupture occur under external loadings? Obviously, this problem is the focus in engineering and the goal of statistical microdamage mechanics.

5.1 Microdamage Number Density. Microdamage, i.e., microcracks and microvoids, usually refers to tiny cracks or voids with sizes comparable to that of the graininess [92,93]. Roughly speaking, microdamage is formed at mesoscopic inhomogeneity, for example, particulates in alloy. So, in polycrystalline metals with grains a few μm in size, the appearance of voids or cracks of about μm in the grain boundaries or within the grains would constitute microdamage nucleation. The density of such microdamages on the surface of metals may be in the range of $10^2 - 10^4/\text{mm}^2$.

Moreover, some microdamage may lead to eventual failure owing to growth and coalescence. Hence, the main issues in damage evolution are three distinctive processes of microdamage: nucleation, growth, and coalescence, and each has its own distinctive mesoscopic kinetics. This means that a trans-scale (from meso- to macroscopic) understanding of damage evolution is badly needed [46,71,72,93].

Curran et al. [93] and Cock and Ashby [94] have given comprehensive discussions on these mesoscopic kinetics. Generally

speaking, there are some common features. Firstly, the mesoscopic kinetics are all dependent on local stress and temperature. Secondly, nucleation usually follows some size distribution, like exponential or Weibull distributions [95]. Thirdly, the growth rate depends on the current size as well as the nucleation size of microdamage. For coalescence, useful predictive models are still not very clear so far. Most importantly, how to correlate these mesoscopic kinetics to the evolution of microdamage number density is a key problem.

As a matter of fact, early in the 1960s, a kind of statistical description of microcracks (i.e., number density of microcracks) was proposed, although the concept was not closely associated with continuum mechanics [96]. Based on actual counts and measurements of microcrack numbers, sizes, and orientations, Curran et al. [93] provided a comprehensive description and called it the concentration of active flaws n_f

$$n_f = n_f(X,t,C,m) \tag{18}$$

where t is time, X is macroscopic Lagrangian coordinate that specifies the center of the material element of interest, and C and m are the size and orientation of microcracks, respectively. Clearly, by simultaneously introducing macroscopic Lagrangian coordinate X and mesoscopic description of microcracks with C, m, and n_f, they intended to introduce the state of microcracks as an internal variable of the constitutive relation in macroscopic formulation of damage evolution. In particular, they made extensive measurement of microcrack population in spallation and put the mesoscopic kinetics of microcracks, such as nucleation and growth rates, into computational codes of continuum mechanics. However, because of lacking the governing evolution equation of microdamage number density in their framework, they had to adopt a priori assumed and unchanged exponential distribution of microcracks in their code,

$$N(X,t,C) = \int_C^{\infty} \int_m n_f(X,t,c',m) dm dc' = N_t(X,t) \exp[-C/C_s(X,t)] \tag{19}$$

where N_t is the total number of microcracks or voids per unit volume and C_s is a characteristic size for the exponential distribution. Under this assumption, they derived that the growth rate of all microcracks should be proportional to their size C. However, this is not in good agreement with the observed and analytic growth rate of microcracks or microvoids. Therefore, it seems that the construction of the equation governing the evolution of the distribution function of microcracks or microvoids is needed.

5.2 Evolution of Microdamage Number Density. Parallel to these, some encouraging approaches to microdamage evolution were proposed [97–99] in which both intrinsic mesoscopic rate processes and statistical features of damage evolution were involved.

Xing [97] suggested that it would be proper to apply the fundamental equation of nonequilibrium statistical physics to the evolution of the distribution function of microcracks,

$$\frac{\partial n(\varepsilon^p,C)}{\partial \varepsilon^p} = q_x(\varepsilon^p)\delta(C-C_0) - \frac{\partial}{\partial C}[C'(\varepsilon^p,C)n(\varepsilon^p,C)] \tag{20}$$

where ε^p is plastic strain, ndC represents the average of microcracks within the size range of C and $C+dC$ in unit volume at time t, q_x and $C'=(dC/d\varepsilon^p)$ are nucleation and growth rates, respectively, δ is Dirac function, and C_0 is nucleation size. As an illustrative example, he made a calculation of microcracks evolution with simple assumptions of $C_0=0$ and C' proportional to $1/\varepsilon^p$.

In order to formulate the evolution of microdamage number density systematically, Bai et al. [98] and Xia et al. [99] examined the element $d\Omega = (p_i, p_i + dp_i)(i=1,2,\ldots)$ in phase space of microdamage, where p_i are all variables describing the state of mi-
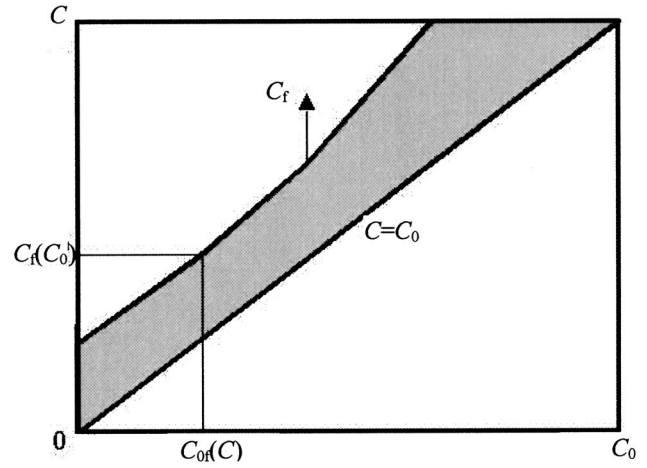

Fig. 4 The solution region of microdamage number density $n_0(t,C,C_0)$. The shaded indicates where nonzero solution locates. $C=C_f(C_0,t)$ or $C_0=C_{0f}(C,t)$ are the microdamage fronts moving upward [114].

crodamage, such as size, orientation, etc., and the balance of microdamage number density in the element owing to nucleation and the flux flowing into and out of the element. This balance leads to the general evolution equation of microdamage number density,

$$\frac{\partial n}{\partial t} + \sum_{i=1}^{I} \frac{\partial(n \cdot P_i)}{\partial p_i} = n_N \tag{21}$$

where t is generalized time, like actual time, nominal deformation, cycles, etc.; n_N is the nucleation rate of microdamage number density, $P_i = \dot{p}_i$, where "·" denotes the rate of variable p_i.

Based on the experimental measurements of mesoscopic kinetics of microdamage, especially in spallation [93,100,101], two most important mesoscopic rate processes, the rates of nucleation n_N and growth $\dot{C}$ of microdamage, are governed by current and initial (nucleation) sizes C and C_0 of microdamage as well as macroscopically local average stress,

$$n_N = n_N(C_0;\sigma) \tag{22}$$

$$V = \dot{C} = V(C,C_0;\sigma) \tag{23}$$

By taking the two general forms of these mesoscopic kinetics, Ke et al. [102], Han et al. [101], and Han and Bai [103] obtained the basic solution to the evolution of microdamage number density n_0 under constant local stress σ in the phase space of $\{C,C_0\}$,

$$n_0(t,C,C_0;\sigma) = \begin{cases} \dfrac{n_N(C_0;\sigma)}{V(C,C_0;\sigma)}, & C \leq C_{f,0} \\ 0, & C \geq C_{f,0} \end{cases} \tag{24}$$

where $C_{f,0}(t;\sigma) = C_f(t,C_0=0;\sigma)$, C_f is the moving front of microdamage with nucleation size C_0 and C_f is determined by $t = \int_{C_0}^{C_f}[dC/V(C,C_0;\sigma)]$, see Fig. 4 [101,102]. In phase space of $\{C\}$, the number density of microdamage n is

$$n(t,C;\sigma) = \int_0^C n_0(t,C,C_0;\sigma) dC_0 \tag{25}$$

The unsteady solution of microdamage number density reveals two fundamental features of microdamage evolution: saturation in the range of small size and an onward movement of microdamage front to larger size. Xia et al. discussed the effects of stochastic growth on the evolution of microdamage. They found that the

major features of microdamage evolution in the stochastic and deterministic models remain the same and the main difference between the two models appears at the vicinity of the saturation boundary [104].

Li and Huang [105] and Li et al. [106] successfully applied the evolution equation of microdamage to the studies of microvoids in ductile metals and craze in glassy polymers, respectively. In terms of the evolution equation of microdamage number density, Hong et al. [107] revealed the multiple peaks in the evolution of short fatigue cracks in metals, and this result is in good agreement with observations.

Now, for the understanding of damage evolution, it seems that the formulation of microdamage number density, which combines the population evolution of microdamage and mesoscopic kinetics in a macroscopic element, has provided a helpful spring board.

In order to form a unified set of the macroscopic equations of mechanics and the kinetics of microdamage number density, it is necessary to define the number density of microdamage in a macroscopic spatial element at macroscopic coordinates $\mathbf{x}$. Then, an associated equations of microdamage evolution and continuum mechanics were given in [99]

$$\frac{\partial n}{\partial t} + \frac{\partial (nA)}{\partial C} + \nabla \cdot (n\mathbf{v}) = n_N \tag{26}$$

$$\frac{\partial \rho_m}{\partial t} + \mathbf{v} \cdot \nabla \rho + \rho \, \nabla \cdot \mathbf{v} = 0 \tag{27}$$

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = \rho^{-1} \nabla \cdot \boldsymbol{\sigma} \tag{28}$$

$$\frac{\partial e_i}{\partial t} + \mathbf{v} \cdot \nabla e_i = \rho^{-1} \boldsymbol{\sigma} : \nabla \mathbf{v} - \rho^{-1} \nabla \cdot \mathbf{h} + q \tag{29}$$

where ρ is mass density, e_i is specific internal energy, $\boldsymbol{\sigma}$ is nominal Cauchy stress tensor, $\mathbf{h}$ is heat flux vector, q is the rate of heat production in unit mass and A is the average growth rate of microdamge with current size C.

It is worth noeting that several similar formulations of microdamage field evolution have been used to a variety of damage-related applications by various researchers. For instance, Van Papenfuss and Muschik [108–110] considered the orientational density of microcracks with various length and direction and formed a corresponding balance equation. Under the assumption of a fixed number of microcracks, some examples with Griffith criterion for growth onset and Mott extension of microcrack were treated and the roles of physical assumptions were discussed.

In the formulation presented by Lemanska et al. [111], the number of crack tips was adopted as the field variable and its evolution is governed by a transport equation. They used the transport equation to the study of crack-tip population in a spherical medium surrounding a charged hole and found some interesting phenomena resulting from the large number of cracks.

Interestingly, another similar formulation was recently applied by Kiselev and Kiselev [112] to the study of superdeep penetration of tungsten particles into a metal target. Superdeep penetration was observed for hard particles with diameter of $<100 \ \mu$m and velocity of $>10^3$ m/s. Then a small fraction of particles can penetrate the target to a depth several hundred or thousand particle diameters. They used a so-called continuum-discrete model to deal with the unusual phenomenon. The particles are described by a collision-free kinetic equation of distribution function of particles in the phase space of macroscopic coordinates and particle velocity. The kinetic equation is combined with the continuum equations of the target. By means of this model, the penetration depth and in-depth distribution of particles in target were calculated and the results are in well agreement with experimental measurements.

5.3 Continuum Damage Based on Microdamage Number Density. As one may note, there are three kinds of coupling in damage evolution: the coupling between damage and stress fields at macroscopic level, the coupling at mesoscopic level, as well as the trans-scale coupling between continuum field and mesoscopic kinetics of microdamage. Hence, a proper closed approximation on macroscopic level but with an appropriate representation of the trans-scale coupling is a key point. In one-dimensional Lagrangian form, the conventional field equations of continuum, momentum, and energy are

$$\frac{\partial \varepsilon}{\partial T} = \frac{\partial v}{\partial X} \tag{30}$$

$$\rho_0 \frac{\partial v}{\partial T} = \frac{\partial \sigma}{\partial X} \tag{31}$$

$$\rho_0 c_v \frac{\partial \theta}{\partial T} = \lambda_h \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial e_{\text{diss}}}{\partial T} \tag{32}$$

where ρ_0 is density of the intact material, λ_h is the heat conductivity, θ is temperature and e_{diss} is the energy dissipated in the material element.

As mentioned previously, we should make the three kinds of coupling be associated with the above three continuum equations. The relation between continuum damage D and the number density of microdamage n is

$$D(t,\mathbf{x}) = \int_0^\infty n(t,\mathbf{x},C) \cdot \tau \cdot dC \tag{33}$$

where τ is the failure volume of an individual microdamage with size C, [99,113,114]. The relation between number density of microdamage and comtinuum damage by averaging was also introduced in [108–110].

Then, the statistical evolution equation of microdamage number density (26) can be converted to the continuum damage field equation by integration under proper boundary conditions [113]

$$\frac{\partial D}{\partial T} + D \frac{\rho}{\rho_0} \frac{\partial v}{\partial Y} = f \tag{34}$$

where

$$f = \int_0^\infty n_N(C;\sigma) \cdot \tau \cdot dC + \int_0^\infty n(t,\mathbf{x},C) \cdot A(C,\sigma) \cdot \tau' \cdot dC \tag{35}$$

f is the dynamic function of damage (DFD), which represents the statistical average effects of nucleation and growth of microdamage on continuum damage evolution and $\tau' = d\tau/dC$. Obviously, the function is an agent bridging mesoscopic kinetics of microdamage and continuum damage.

Now, the five field equations (30)–(34) form an associated system for the coupled trans-scale framework. In this formulation, damage D is no longer an internal variable in constitutive law, but is a field variable coupled with other field variables. On the other hand, through this variable continuum field is coupled to mesoscopic kinetics of microdamage. Hence, this associated system of equations is different from the conventional continuum mechanics, where the three continuum field equations are decoupled to any micro- or mesoscopic kinetics of media by means of constitutive relations.

Under the assumption of small damage and locality of constitutive relations, the substitution of the obtained solution of microdamage number density n (24) into integral (35) leads to a closed DFD without microdamage number density n and expressed directly by two mesoscopic kinetic laws of nucleation and growth rates of microdamage [114],
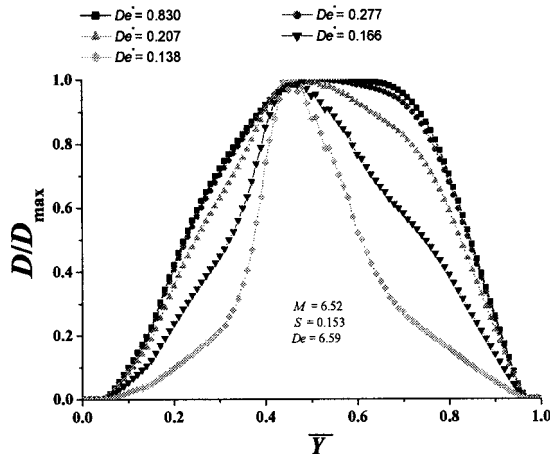
Fig. 5 Effects of De* on damage localization trans-scale sensitivity [116]

$$f = \int_0^\infty n_N(C;\sigma)\tau(C)dC\left\{1 + \frac{\int_0^\infty n_N(C_0;\sigma)\int_{c_0}^{c_f}\tau'(C)dCdC_0}{\int_0^\infty n_N(C;\sigma)\tau(C)dC}\right\} \quad (36)$$

The expression of (36) indicates that DFD consists of two parts: a time-independent term governed by nucleation rate only and a time-dependent one governed by both nucleation and growth rates of microdamage. This provides a physical interpretation of the concept of simple and compound damages proposed by Davison and Stevens [44].

The advantage of this approximate, closed trans-scale formulation is that it forms a unified set of the macroscopic equations of mechanics and the kinetic equations of microdamage, and the set can be solved simultaneously, as Barenblatt suggested [72]. Wang et al. [115,116] used this trans-scale formulation to fulfill the calculation of the process of spallation, directly based on the knowledge of mesoscopic kinetics of nucleation and growth of microdamage. Figure 5 shows the damage profiles with varing De* and the emergence of localized damage. Moreover, by means of this trans-scale formulation, Wang et al. [116] were able to analyze the energy dissipation due to microdamage evolution and explained why spallation is mainly a rate-dependent phenomenon and not sensitive to macroscopic energy dissipation, as experiments and empirical formula demonstrated.

Provided strain and damage are both negligibly small and strain and damage rates are in the same order, the damage field equation (34) becomes

$$\frac{\partial D}{\partial T} = \dot{D} \approx f \quad (37)$$

This is what internal variable theory in continuum damage mechanics assumed [11]. Obviously, the expression (36) is the evolution law in continuum damage mechanics.

Therefore, this trans-scale approximation not only provides a physical ground of continuum damage mechanics, but also gives an expression of damage evolution law in terms of mesoscopic kinetics directly.

6 Mechanisms Underlying Trans-Scale Coupling

The mechanisms governing the enhancement of the effect of mesoscopic length scales on macroscopic properties may vary from case to case. This is the most tricky aspect in the analysis of

coupled multiple length and time scales. As pointed out by Bazant and Chen [3], the scaling problem of main interest in solid mechanics is the dependence of nominal strength on its macroscopic structure size (denoted by L). In most cases, as discussed in Sec. 2, the mesoscopic length scale l is usually much less than the structure size L, like the specimen size in the laboratory. That is to say, the ratio

$$R = \frac{l}{L} \ll 1 \quad (38)$$

Then, how can the small mesoscopic length scale be coupled to macroscopic length scales and then affect macroscopic behaviors?

6.1 Effects of Total Mesoscopic Elements. As discussed in Sec. 3, the straightforward way for mesoscopic elements to affect macroscopic behavior is the total of mesoscopic elements. Plasticity resulting from dislocation motion, characterized by the dislocation Burgers vector, is a most impressive example. In the dislocation plasticity theory proposed by Orowen and Taylor, there is only one microscopic length scale, the Burgers vector $\mathbf{b}$, with no business of macroscopic length scales, like the size of specimen.

As a matter of fact, the macroscopic strain rate $\dot{\gamma}$ can be derived in the following way [46],

$$\dot{\gamma} = \frac{u}{Lt} = \frac{N_d\mathbf{b}}{Lt} = \frac{N_d\mathbf{b}}{L^2}\frac{L}{t} = \rho_d\mathbf{b}v_d \quad (39)$$

where N_d is the number of dislocations. Hence, the effect of the small microscopic length scale, the Burgers vector $\mathbf{b}$, is enhanced to the macroscopic strain rate by the total of dislocation on a unit area. Clearly, this is a trivial way for trans-scale enhancement.

In one word, there are two different physics: dislocation motion on microscopic level and shear strain on macroscopic level, respectively. The mechanism to relate these two processes is the effect of the total of dislocations.

6.2 Competing Macroscopic and Mesoscopic Time Scales. Once a problem is rate dependent, namely, multiple time scales are involved in the problem, we may have to understand how to characterize the coupled trans-scale effects of multiple length and time scales.

Since the mechanisms governing the trans-scale coupling effects vary from case to case, it is truly important to concretely demonstrate how the multiscales are coupled and how the coupled combination plays the crucial role in bridging various length and time scales by case study. To illustrate these, we go back to the problem of spallation, a real multiscale and rate-dependent problem (see Wang et al. [116]).

Since the continuum mass and momentum equations are not directly related to the trans-scale formulation in spallation, here we list the dimensionless equations of energy and damage evolution only [116]

$$\frac{\partial\bar{\theta}}{\partial\bar{T}} = \bar{\sigma}\frac{\partial\bar{\varepsilon}^p}{\partial\bar{T}} + D^*\frac{\partial\bar{\Sigma}}{\partial\bar{T}} + \Psi\frac{\partial^2\bar{\theta}}{\partial\bar{Y}^2} \quad (40)$$

$$\frac{\partial\bar{D}}{\partial\bar{T}} + \mu\frac{\varepsilon^*}{1+\varepsilon^*\bar{\varepsilon}}\bar{D}\frac{\partial\bar{v}}{\partial\bar{X}} = \frac{1}{\text{De}^*}\cdot\bar{f}(D^*,\bar{D},\bar{\sigma}) \quad (41)$$

All variables with bars are dimensionless and normalized, i.e., in $O(1)$.

The dimensionless numbers are defined as follows:
The intrinsic Deborah number:

$$D^* = \frac{n_N^* c^{*5}}{V^*} \quad (42)$$

The imposed Deborah numbers:

$$\mathrm{De}^* = \frac{ac^*}{LV^*}$$

$$\text{or } \mathrm{De} = \frac{a}{Ln_N^* c^{*4}}, \tag{43}$$

In Eqs. (40)–(43), a is the elastic wave speed, ε^* is the strain corresponding to the threshold stress σ^*, $\varepsilon^* = \sigma^*/E$, $E = \rho_0 a^2$ is the elastic modulus of the material, $\Sigma = \alpha_\Sigma \int_0^\infty n C^2 dC = (n_N^* C^{*4}/V^*) O(1) = D^* O(1)$ is the corresponding surface of damage.

Contrary to common sense, the ratio of length scales on the meso- and macro-levels $R = C^*/L$ does not appear in the governing equations. This looks very abnormal at first sight. Actually, the imposed Deborah number De^* is a combination of two ratios: the size scale ratio C^*/L and the ratio of two velocities V^*/a. De^* is the unique trans-scale dimensionless parameter because the elastic wave speed a and the sample size L are macroscopic parameters, whereas microdamage size C^* and microdamage growth rate V^* are mesoscopic ones. This is very different from all other dimensionless parameters. On the other hand, $\mathrm{De}^* = t_V/t_{\mathrm{im}}$ refers to the ratio of microdamage growth time scale $t_V = C^*/V^*$ over the macroscopically imposed time scale $t_{\mathrm{im}} = L/a$. Hence, it represents the competition and coupling between the macroscopically imposed wave loading and the intrinsic microdamage growth. In the concerned case $\mathrm{De}^* < 1$, this means that microdamage has enough time to grow during the macroscopic wave loading. Thus, the microdamage growth may be the predominate mechanism governing spallation.

The intrinsic Deborah number D^* represents the relation of the two mesoscopic kinetics: microdamage growth and nucleation. However, for the intrinsic Deborah number D^*, there are four points worth emphasizing. First, D^* characterizes the rate ratio of two intrinsic mesoscopic processes: nucleation over growth. Actually, $D^* = t_v/t_N$, where $t_V = C^*/V^*$ and $t_N = (n_N^* C^{*4})^{-1}$ are the growth and nucleation time scales, respectively. Second, the intrinsic Deborah number D^* implies a certain characteristic damage, since

$$D(t,\mathbf{x}) = \int_0^\infty n(t,\mathbf{x},C) \cdot \tau \cdot dC = \frac{n_N^* C^{*5}}{V^*} \int_0^\infty \bar{n}_N d\bar{C}_0 \int_{\bar{C}_0}^{\bar{C}_f} \frac{\bar{C}^3}{\bar{V}} d\bar{C} \tag{44}$$

In this sense, the roles of mesoscale C^* and nucleation density rate n_N^* in continuum damage look like the combination of Burgers vector $\mathbf{b}$ and dislocation density ρ_d in plasticity. Third, it has been derived that D^* is a proper indicator of macroscopic critical damage to localization [114,117]. In fact, it is found in simulations that the critical damage to localization in spallation is about $(10^{-3}–10^{-2})$, i.e., in the order of D^*. Finally, this small D^* indicates that the energy dissipation due to microdamage is negligible compared to the bulk plastic work. This explains why spallation cannot be formulated by macroscopic energy criterion but must be treated by multiscale analysis [115,116].

In one word, in the case study of spallation, there are three physical processes with three time scales at two levels: the macroscopic imposed time scale $t_{\mathrm{im}} = L/a \sim 10^{-6}$ s and two mesoscopic time scales, growth time scale $t_v = C^*/V^* : 10^{-6}$ s and the nucleation time scale $t_N = (n_N^* C^{*4})^{-1} : 10^{-3}$ s. They can form two independent Deborah numbers, which govern the trans-scale coupling. This means that the competition of the three rates is the underlying mechanism. Moreover, this mechanism of rate competition and coupling can interpret why the very short atomic processes with frequency of $10^{12\text{-}13}$ Hz is less important than microdamage and can be represented by their averages at mesoscale as kinetics of microdamage in macroscopic damage accumulation.

6.3 Balancing Macroscopic and Mesoscopic Diffusions.
Different from the Deborah numbers governing spallation, in the previously cited formulation of damage accumulation (17), given by Barenblatt [72], the macroscopically effective diffusion of damage is

$$\kappa_L \sim \frac{L^2}{\tau_{\mathrm{im}}} : \left[\frac{\lambda^2 \sigma_s}{\tau_r(1-D)} \frac{\partial q_b}{\partial \sigma_s} \right] \tag{45}$$

where τ_{im} is the macroscopic characteristic time scale. So, the dimensionless parameter governing the trans-scale coupling in the formulation is

$$\frac{\lambda^2/\tau_r}{L^2/\tau_{\mathrm{im}}} : \frac{\kappa}{\kappa_L} \tag{46}$$

This implies that the macroscopic and mesoscopic diffusions should be on the same order, or the imposed characteristic time scale τ_{im} will be $(L/\lambda)^2$ times the mesoscopic relaxation time scale τ_r of microdamage. This might be the reason that the model is applied to creep.

In one word, there are two different diffusions at mesoscopic and macroscopic levels, respectively. Though they have different relaxation time scales, the mechanism underlying the trans-scale coupling is their balancing diffusion coefficients.

6.4 Emergent Structures.
There are some other distinctive types of problems in solids with coupled multiple length and time scales, in which the emergence of new structures with distinctive length scales plays a significant role in trans-scale coupling [118]. This may be difficult but fascinating. For example, with chemical reaction and various diffusions (i.e., mass diffusion, viscosity, momentum diffusion, and energy or heat diffusion), multiscale and rate-dependent processes become much more complicated. Apart from geometric and dynamic similarities, chemical similarity should be taken into account. Chemical similarity alludes to maintaining the same mass transfer and chemical reaction characteristics as in the smaller system. One of the most important characteristic dimensionless numbers in this field is the Thiele modulus ϕ [119].

The Thiele modulus ϕ describes the relative importance of surface reaction relative to their ability to diffuse

$$\phi = r_0 \sqrt{\frac{K C_{\mathrm{Ai}}^{m_r-1}}{D_i}} \tag{47}$$

where r_0 is the radius of a spherical particle, K is reaction rate constant, C_{Ai} is surface concentration, D_i is diffusion coefficient, and m_r is the order of reaction. Clearly, in the Thiele modulus, there is a mesoscopic length scale, i.e., the individual particle radius r_0, but not macroscopic size L. However, the Thiele modulus implies some emergent length scale, which does not exist initially. As a matter of fact, the definition of the Thiele modulus ϕ indicates the competition of the mesoscopical particle size r_0 and an emergent length scale l_e of chemical reaction,

$$l_e = \sqrt{\frac{D_i}{K C_{\mathrm{Ai}}^{m_r-1}}} \tag{48}$$

The emergence of new mesoscopic structures is by no means a rare event, but a frequent issue in engineering. The emergence of thermoplastic shear banding is the other example of the emergent length scale in solid mechanics [120],

$$l_e : \sqrt{\frac{\lambda_h \cdot \Delta\theta}{\tau \dot{\gamma}}} \tag{49}$$

where λ_h is the heat conductivity of solid, and τ and $\dot{\gamma}$ are shear stress and shear strain rate, respectively.

It seems that the appearance of localized structure with emergent length scale may be a common feature in generalized

reaction-diffusion systems, and the square of the new emergent length scale appears to be proportional to the diffusion but inversely proportional to the reaction or a source term [121].

In one word, the balance of the emergent structure and the existing ones may become a mechanism underlying the trans-scale coupling.

Above all, the four examples reviewed in this section can by no means embrace all possible mechanisms governing the trans-scale coupling in problems with multiple space and time scales. As we stressed again and again, the mechanisms underlying multiphysics and multiscale coupling may vary from case to case. Hence, to explore the balance and competition of various physics at various levels is a real challenge and a fascinating job [122].

7 Evolution-Induced Catastrophe and Trans-Scale Sensitivity

"The world of many particle systems, which is the primary focus of statistical theory, is so diverse and rich that nobody could guess the richness that it contains using just pure thought or pure theory," [64]. Evolution induced catastrophe in solids is such a subject [71].

The above formulation of statistical microdamage mechanics can help us to deal with damage evolution to catastrophe, but it does not provide insight into the very catastrophe transition. This implies a need of some new concepts on the nonequilibrium and strong interaction between microdamage when a system is approaching failure and multiscale correlations occur from mesoscopic to macroscopic scales in the system.

Actually, the catastrophic failure in solids results from a cascade of microdamage from small to large scales in the case far from equilibrium. For such a complicated process, experiment and/or simulation have to come first to expose the main phenomena. In fact, any research study of statistical phenomena must be heavily based on experimental knowledge. In the exploration, one is looking into uncharted territory hoping to find something new and unexpected [64].

7.1 Sample Specificity. Clearly, the catastrophic failure in solids is sensitively dependent on some details of mesoscopic heterogeneity and dynamical evolution of damage plays an important role in the catastrophe transition. Recent numerical simulations [27,67,68] show that the σ-ε data collapse onto the same curve in the initial regime. But, with increasing deformation, the deviations from the scaling become stronger and stronger and differences in failure of samples become distinct. These results reveal that the samples show a transition from universal scaling to sample-specific behavior with increasing damage, namely, significant differences at failure from sample to sample, even though they are initially identical macroscopically. This leads to an important feature of evolution induced catastrophe: sample specificity (SS) [71,123].

This sample specificity in evolution-induced catastrophe is very common in engineering. According to their lattice simulation on failure of fiber composites, Curtin [25] and Curtin and Scher [26] co-workers indicated the complexity of failure. "Distributed fiber failure continues in a stable manner until a cluster of broken fibers reaches some critical size, triggering the catastrophic failure of the composite." And, "a fully predictive capability is presently lacking because the statistical characteristics of the fiber strength distribution require a 3D calculation with the features of fiber pull-out, accurate load transfer functions from broken to intact fibers, and resulting stress distributions for arbitrary spatial distributions of the fiber breaks."

7.2 Trans-Scale Sensitivity. As a nonlinear dynamical system, the final state is determined by its initial state, but the final differences at failure between the samples with minor initial differences at mesoscopic level can be strongly enhanced by the dynamical evolution. Thus, both initial mesoscopic differences and nonlinear dynamical evolution are inherent in the sample
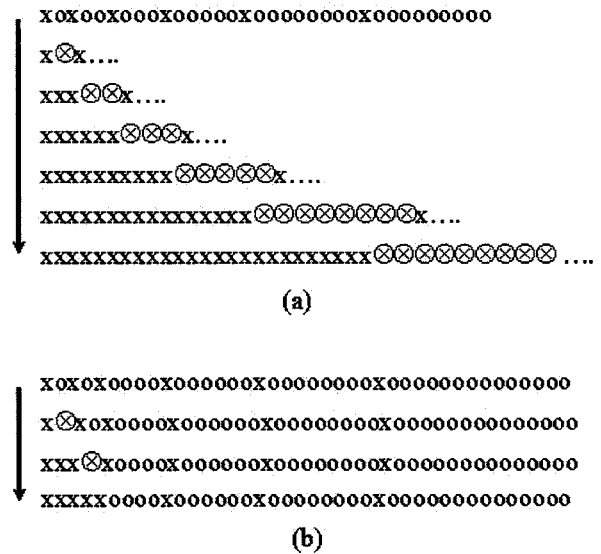
(a)

(b)

Fig. 6 (*a*) **Quasi-Fibonacci series (1,2,3,5,8,13,20,...) of breaking is a sensitive microstructure for cascading, In (*b*), the cascading will stop at a stable status [71].**

specificity of failure. This is a trans-scale sensitivity [71,123–125]. The ideas of sample specificity and trans-scale sensitivity can be illustrated by a simple evolution of two chains with the same initial damage fraction but a minor difference in initial damage pattern, i.e., Hamming distance tends to zero (see Fig. 6). In fact, this simple example clearly demonstrates that though the two samples are the same in initial damage fraction (an average at macroscopic level), nearly the same in distribution function (a statistical description of mesoscopic elements), even in Hamming distance when N tends to infinity (an exact static description of patterns at mesoscopic level), they still behave differently in the cascade of microdamage evolution to catastrophe. This indicates the significance of trans-scale sensitivity at catastrophe transition.

Sahimi and Arbabi [126] emphasized the importance of this phenomenon clearly. "In real engineering materials and in natural rocks, the presence of large number of flaws with various sizes, shapes, and orientations makes the problem far more complex. Disorder comes into play in many ways during a fracture process. Even small, initially present disorder can be enormously amplified during fracture. This makes fracture a collective phenomenon in which disorder plays a fundamental role. In fact, due to disorder brittle materials generally exhibit large statistical fluctuations in fracture strengths, when nominally identical samples are tested under identical loading. Because of these statistical fluctuations, it is insufficient, and indeed, inappropriate, to represent the fracture behavior of a disordered material by only its average properties, an idea which is usually used in mean-field approaches: fluctuations are important and must not be neglected."

7.3 Emergence of Failure Structure. In principle, the failure of brittle media can be formulated by traditional energy criterion, like the instability of rock samples under an elastic testing machine [127]. A catastrophe can be defined as the infinitesimal increment of external controlling variable that induces finite response of the system. In particular, a catastrophe occurs when released and dissipative energy become larger than the external input work. However, a catastrophe may occur at different scales. The catastrophe at mesoscopic scale is usually attributed to mesoscopic damage, whereas the materials failure corresponds to the catastrophe at macroscopic or global scale. Unfortunately, when the media are no longer homogeneous, namely, heterogeneous

with mesoscopic structures, the energy criterion may become uncertain. Zhang et al. provide an example to illustrate the interesting aspect of statistical microdamage mechanics [128].

For a heterogeneous elastic brittle medium with Weibull distribution of mesoscopic strength, under local mean field approximation, its nominal stress σ_0–strain ε relation can be expressed by

$$\sigma_0 = \varepsilon \exp(-\varepsilon^\beta) \tag{50}$$

Beyond the maximum of nominal stress, any mesoscopic elements of the sample may deform either with increasing strain or with decreasing strain under the same nominal stress. So, in this situation, the sample may be split into two parts. In one part, damage will no longer develop and deformation will restore, correspondingly their stored elastic energy will release. In the other part, the material will continue its deformation and damage process, supported by the energy input into the part. Provided the released elastic energy exceeds the energy required by the continuing damage part of the sample, a self-sustained failure may occur, even though the external boundary condition remains rigid, namely, no external input energy at all. This critical condition can be expressed by

$$\varepsilon_f \xi = \varepsilon_{\sigma \max} \tag{51}$$

where ε_f is the critical strain for failure, $0 \leqslant \xi = [(L_0 - L_f)/L_0]^{1/\beta} \leqslant 1$ is the size ratio, and L_0 and L_f are the sizes of the sample and the failure part, respectively.

From this simple analysis, the critical strain to failure may vary from the strain at maximum stress to infinity, depending on the size ratio ξ. Actually, this is an uncertainty relation between the critical strain to failure and the size ratio. The reason for the uncertainty lies in that without thorough and exact knowledge of the mesoscopic details of the sample; one cannot know which part of the sample will fail beforehand. In fact, unless knowing all mesoscopic information, like the mesoscopic spatial distribution of heterogeneity and the induced stress fluctuation in the sample, one can never predict the failure accurately.

7.4 Critical Sensitivity (CS). "In breakdown process, such as the seemingly simple case of brittle fracture—i.e., fracture that do not involve local plastic deformation—extremely sensitivity to rare events makes the problem very difficult to handle theoretically" [53]. In fact, owing to sample specificity, trans-scale sensitivity, and the emergence of the failure structure in evolution-induced catastrophe, we can hardly obtain a deterministic prediction of failure in inhomogeneous media, by either macroscopic averaging parameters or conventional statistics of mesoscopic inhomogeneities. Then what can we do in prediction of failure in such solids? Perhaps, the concept of critical sensitivity can provide some help in coping with the problem [71,125,128–130].

The critical sensitivity means that, in heterogeneous brittle media, the response of the system to controlling variables, such as external loading, may become significantly sensitive as the system approaches its catastrophe transition point. There may be various definitions of the response and sensitivity. For example, the sensitivity can be defined by [71,125]

$$S = \frac{\Delta E'/\Delta \sigma'}{\Delta E/\Delta \sigma} \tag{52}$$

where $\Delta \sigma$ and $\Delta \sigma'$ are the increments of the load (i.e., the governing external variables) and $\Delta \sigma\prime$ is greater than $\Delta \sigma$, ΔE and $\Delta E'$ are the increments of energy release induced by $\Delta \sigma$ and $\Delta \sigma'$, respectively. It has been found that there is a significant increase in S when a sample is approaching its catastrophe transition (see Fig. 7). Hence, if the increments of energy release and a governing variable are both measurable, critical sensitivity may provide clues for catastrophe prediction. This concept has been applied to
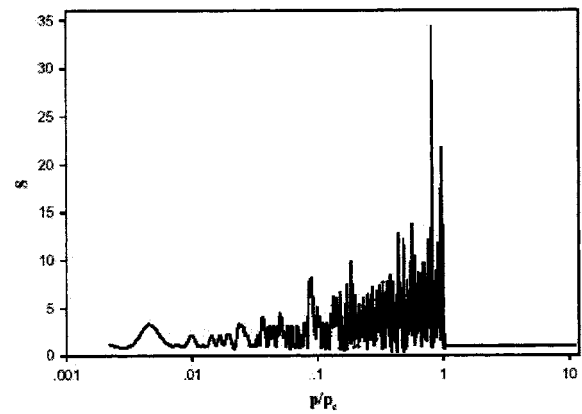

Fig. 7 Critical sensitivity of cluster mean field (CMF) model (N=10,000, β=2) [125]

rock tests and earthquake forecasts and looks promising [128].

Finally, we have to point out that the above-reviewed concepts, though very helpful in understanding the trans-scale coupling at failure, are by no means a complete formulation of evolution-induced catastrophe. More effective and practical formulations are still sorely needed.

8 Concluding Remarks

From this review, the following remarks can be made.

The study of phenomena with coupled multiple space and time scales is a need and an opportunity. This is especially true for the problems in solid mechanics with microstructures. In particular, stiffness reflects the average behavior of microstructure, whereas strength is very sensitive to the microstructural details. Therefore, the main concern of coupled multiple space and time scales in solid mechanics should be devoted to those topics related to breakdown properties. In order to form a trans-scale theoretical framework to link coupled multiple space and time scales, the mechanisms governing how the mesoscopic kinetics are in balance with macroscopic equations of mechanics should be clarified.

The fundamental difficulty in the problems with coupled multiple length and time scales in solid mechanics is due to the hierarchy and evolution of microstructures with various physics and rates at various length levels in solids. These multiscale problems are different from those relatively uniform and with physical similarity on various scales or with weak coupling on two levels. Thus, global averaging, similarity solutions, or perturbation methods are no longer suitable; some new statistical approaches should be explored.

The microstructures at mesoscales in solids are various and complicated. Unlike the kinetic theory of gas and the theory of linear oscillators of solids, the entity and representation in statistical mesomechanics of solids can vary greatly, depending on various problems. Some typical frameworks are reviewed in this paper. The emphasis is put on their representations, characteristic mesoscopic length scales, the mechanisms governing the transfer of mesoscale parameters to macrobehavior, and their trans-scale formulation.

For trans-scale damage evolution, usually there are several length and time scales. The length scales are microdamage size at mesoscale and the sample size at macrolevel, whereas the time scales are nucleation and growth rates of microdamage at mesolevel and the imposed loading duration at macrolevel. So, statistical mesomechanics with the associated equations of continuum mechanics and microdamage evolution is a proper approach, which includes the nonlinear coupling of the stress field and the evolution of microdamage. For an illustrative problem—

spallation, the Deborah numbers, namely, the ratios of multiple time scales—are the key factors governing the multiscale process.

A cascade of damage evolution magnifies the effects of microstructures on failure and induces trans-scale sensitivity. This is the essence of sample specificity (SS) in failure. For the sake of predicting evolution-induced catastrophe, the concept of critical sensitivity seems to be promising, in practice.

The new challenge in statistical mesomechanics of solids is to deal with the coupled physics with multiple time scales at multiple space scales, to understand the nonequilibrium evolution to catastrophe of engineering significance, to establish the corresponding coupled trans-scale formulations, and to clarify the mechanisms underlying the trans-scale coupling

Acknowledgment

References

[1] Tsien, S. H., 1962, *Physical Mechanics*, Science Press, Beijing (in Chinese).
[2] Marder, M., and Fineberg, J., 1996, "How Things Break," Phys. Today, **49**(9), pp. 24–29.
[3] Bazant, Z. P., and Chen, E. P., 1997, "Scaling of Structural Failure," Appl. Mech. Rev., **50**(10), pp. 593–627.
[4] Becker, K. C., Byington, C. S., Forbes, N. A., and Nickerson, G. W., 1998, "Predicting and Preventing Machine Failures," Ind. Phys., **4**(4), pp. 20–23.
[5] Miller, K. J., 1999, "A Historical Perspective of the Important Parameters of Metal Fatigue and Problems for the Next Century," *Fatigue 99 Proc. 7th Int. Fatigue Congress*, X. R. Wu and Z. G. Wang, eds., Higher Education Press, Beijing, pp. 15–40.
[6] Glimm, J., and Sharp, D. H., 1997, "Multiscale Science: A Challenge for the Twenty-First Century," SIAM News, **30**(8), pp. 1–7.
[7] Eshelby, J. D., 1957, "The Determination of the Elastic Field of an Ellipsoidal Inclusion and Related Problems," Proc. Phys. Soc., London, Sect. A, **A241**(1226), pp. 376–396.
[8] Mura, T., 1982, *Micromechanics of Defects in Solids*, Martinus Nijhoff Publishers, The Hague.
[9] Budiansky, B., 1983, "Micromechanics," *Advances and Trends in Structural and Solid Mechanics*, A. K. Noor and J. M. Housner, eds., Pergamon Press, Oxford, pp. 3–12.
[10] Kachanov, L. M., 1986, *Introduction to Continuum Damage Mechanics*, Matinus Nijhoff, Dordrecht.
[11] Lemaitre, J., 1992, *A Course on Damage Mechanics*, Springer-Verlag, Heidenberg.
[12] Chaboche, J. L., 1988, "Continuum Damage Mechanics, I General Concepts," ASME J. Appl. Mech., **55**(1), pp. 59–64.
[13] Krajcinovic, D., 1998, "Selection of Damage Parameter: Art or Science?," Mech. Mater., **28**(1–4), pp. 165–179.
[14] Haritos, G. K., Hager, J. W., Amos, A. K., Salkind, M. J., and Wang, A. S. D., 1988, "Mesomechanics: The Microstructure-Mechanics Connection," Int. J. Solids Struct., **24**(11), pp. 1081–1096.
[15] E, Weinan, and Engquist, B., 2003, "The Heterogeneous Multiscale Methods," Commun. Math. Sci., **1**(1), pp. 87–132.
[16] Brandt, A., 2001, "Multiscale Scientific Computation: Review 2001," *Multiscale and Multiresolution Methods: Theory and Applications*, T. J. Barth, T. F. Chan, and R. Haimes, eds., Springer Verlag, Heidelberg.
[17] Li, J., and Kwauk, M., 2003, "Exploring Complex Systems in Chemical Engineering—The Multi-Scale Methodology," Chem. Eng. Sci., **58**(3–6), pp. 521–535.
[18] Chong, L., and Ray, L. B., 2002, "Whole-Istic Biology," Science, **295**(5560), p. 1661.
[19] Bulatov, V., Cleri, F., Colombo, L., Lewis, L., and Mousseau, N., 2001, *MRS Proc. on Advances in Materials Theory and Modeling: Bridging Over Multiple-Length and Time Scales*, Vol. 677, MRS, Warrendale, PA.
[20] Needleman, A., 2000, "Computational Mechanics at the Mesoscale," Acta Mater., **48**(1), pp. 105–124.
[21] Mishnaevsky, L. L., and Schmauder, S., 2001, "Continuum Mesomechanical Finite Element Modeling in Material Development: A State of-the-art Review," Appl. Mech. Rev., **54**(1), pp. 49–67.
[22] Cui, J. Z., and Yang, H. Y., 1996, "A Dual Coupled Method of Boundary Value Problems of PDE With Coefficients of Small Period," Int. J. Comput. Math., **14**(2), pp. 159–174.
[23] Cui, J. Z., Shin, T. M., and Wang, Y. L., 1999, "The Two-Scale Analysis Method for Bodies With Small Periodic Configurations," Struct. Eng. Mech., **17**(6), pp. 601–614.
[24] Cuitino, A. M., Stainier, L., Wang, G., Strachan, A., Cagin, T., Goodard, W. A., III, and Ortiz, M., 2001, "A Multiscale Approach for Modeling Crystalline Solids," J. Comput.-Aided Mater. Des., **8**(2–3), pp. 127–149.
[25] Curtin, W. A., 1999, "Stochastic Damage Evolution and Failure in Fiber-Reinforced Composites," Adv. Appl. Mech., **36**, pp. 164–248.
[26] Curtin, W. A., and Scher, H., 1997, "Time-Dependent Damage Evolution and Failure in Materials I Theory," Phys. Rev. B, **55**(18), pp. 12038–12050.
[27] Herrmann, H. J., and Roux, S., 1990, *Statistical Models for the Fracture of Disordered Media*, North-Holland, Amsterdam.
[28] Meakin, P., 1991, "Models for Material Failure and Deformation," Science, **252**(5003), pp. 226–234.
[29] Liang, N. G., Liu, H. Q., and Xu, H. Q., 1997, "A Multi-Scale Network Model and Parameter Optimization of Discontinuous Fiber Reinforced Composites," *Macro-Micro/Meso Mechanical Properties of Materials*, M. Tokuda and B. Y. Xu, eds., Mie Academic Press, pp. 269–275.
[30] Hansen, A., Hinrichsen, E. L., and Roux, S., 1991, "Roughness of Crack Interfaces," Phys. Rev. Lett., **66**(19), pp. 2476–2479.
[31] Schmittbuhl, J., Hansen, A., and Batrouni, G. G., 2003, "Roughness of Interfacial Crack Fronts: Stress-Weighted Percolation in the Damage Zone," Phys. Rev. Lett., **90**(4), pp. 045505.
[32] Hansen, A., and Schmittbuhl, J., 2003, "Original of the Universal Roughness Exponent of Brittle Fracture Surfaces: Stress-Weighted Percolation in the Damage Zone," Phys. Rev. Lett., **90**(4), pp. 045504.
[33] Panin, V. E., 1998, "Foundation of Physical Mesomechanics," Phys. Mesomechanics, **1**(1), pp. 5–20.
[34] SIAM Journal on Multiscale Modeling and Simulation.
[35] Sih, G. C., 2000, *Mesomechanics 2000 Role of Mechanics for Development of Science and Technology*, G. C. Sih, ed., Tsinghua University Press, Beijing, China.
[36] Pyrz, R., Schjødt-Thomsen, J., Rauhe, J. C., Thomsen, T., and Jensen, L. R., 2002, *New Challenges in Mesomechanics 2002*, R. Pyrz, J. Schjødt-Thomsen, J. C. Rauhe, T. Thomsen, and L. R. Jensen, eds., Aalborg University, Denmark.
[37] Sih, G. C., and Spyropoulos, C. P., 2002, *Inter. Symp. of Multiscaling in Mechanics*, G. C. Sih and C. P. Spyropoulos, eds., National Technical University of Athens Press, Athens.
[38] Tuler, F. R., and Butcher, B. M., 1968, "A Criterion for the Time Dependence of Dynamic Fracture," Int. J. Fract. Mech., **4**(4), pp. 431–437.
[39] Shen, L. T., Zhao, S. D., Bai, Y. L., and Luo, L. M., 1992, "Experimental Study on the Criteria and Mechanism of Spallation in an Al Alloy," Int. J. Impact Eng., **12**(1), pp. 9–19.
[40] Zhu, Z. X., 1985, private communication.
[41] Dremin, A. N., Molodets, A. M., Melkumov, A. I., and Kolesmikov, A. V., 1992, "On Anomalous of Steel Spall Strength and its Relationship to Martinsitic Transformation," *Shock Wave and High Strain Rate Phenomena in Materials*, M. A. Meyers, L. E. Murr, and K. P. Staudhammer, eds., Dekker, New York, pp. 751–757.
[42] Kanel, G. I., Rasorenov, S. V., and Fortov, V. E., 1992, "The Dynamic Strength of Copper Single Crystals," *Shock Wave and High Strain Rate Phenomena in Materials*, M. A. Meyers, L. E. Murr, and K. P. Staudhammer, eds., Dekker, New York, pp. 775–782.
[43] Kanel, G. I., Rasorenov, S. V., and Utkin, A. V., 1995, "Spallation in Solids Under Shock-Wave Loading: Analysis of Dynamic Flow, Methodology of Measurements, and Constitutive Factors," *High-Pressure Shock Compression of Solids II: Dynamic Fracture and Fragmentation*, L. Davison, D. E. Grady, and M. Shahinpoor, eds., Springer-Verlag, New York, pp. 1–24.
[44] Davison, L., and Stevens, A. L., 1972, "Continuum Measures of Spall Damage," J. Appl. Phys., **43**(3), pp. 988–994.
[45] Davison, L., and Stevens, A. L., 1973, "Thermomechanical Constitution of Spalling Elastic Bodies," J. Appl. Phys., **44**(2), pp. 668–674.
[46] Meyers, M. A., 1994, *Dynamic Behaviour of Materials*, Wiley, New York.
[47] Grady, D. E., and Kipp, M. E., 1993, "Dynamic Fracture and Fragmentation," *High-Pressure Shock Compression of Solids*, J. R. Asay and M. Shahinpoor, eds., Springer-Verlag, New York, pp. 265–332.
[48] Grady, D. E., 1988, "The Spall Strength of Condensed Matter," J. Mech. Phys. Solids, **36**(3), pp. 353–384.
[49] Clifton, R. J., 2000, "Response of Materials Under Dynamic Loading," Int. J. Solids Struct., **37**(1–2), pp. 105–113.
[50] Voyiadjis, G. Z., Palazotto, A. N., and Gao, X. L., 2002, "Modeling of Metallic Materials at High Strain Rates With Continuum Damage Mechanics," Appl. Mech. Rev., **55**(5), pp. 481–493.
[51] Hayhurst, D. R., Dimmer, P. R., and Morrison, C. J., 1984, "Development of Continuum Damage in the Creep Rupture of Notched Bars," Philos. Trans. R. Soc. London, Ser. A, **A311**(1516), pp. 103–129.
[52] Field, J. E., Swallowe, G. M., and Heavens, S. N., 1982, "Ignition Mechanisms of Explosives During Mechanical Deformation," Proc. R. Soc. London, Ser. A, **A382**(1782), pp. 231–244.
[53] Hansen, A., Hinrichsen, E. L., and Roux, S., 1991, "Scale-Invariant Disorder in Fracture and Related Breakdown Phenomena," Phys. Rev. B, **43**(1B), pp. 665–678.
[54] Bochenek, B., and Pyrz, R., 2002, "Reconstruction Methodology for Planar and Spatial Random Microstructure," *New Challenges in Mesomechanics 2002*, R. Pyrz, J. Schjødt-Thomsen, J. C. Rauhe, T. Thomsen, and L. R. Jensen, eds., Det Obelske familiefond, Denmark.
[55] Reichl, L. E., 1980, *A Modern Course in Statistical Physics*, University of Texas Press, Austin.
[56] McDowell, D. L., 1997, "Applications of Continuum Damage Mechanics to Fatigue and Fracture," STP 1315, pp. 1–3.
[57] Horie, Y., Davison, L., and Thadhani, N. N., 2002, *High-Pressure Shock Compression of Solids VI*, Springer, New York.

[58] Sih, G. C., 2002, "Preface," *Int Symp of Multiscaling in Mechanics* G. C. Sih and C. P. Spyropoulos, eds., National Technical University of Athens Press, Athens.

[59] Pantelides, S. T., 1992, "What is Materials Physics, Anyway?," Phys. Today, **45**(9), pp. 67–69.

[60] Zhao, K. H., 1991, *Qualitative and Quasi-Quantitative Physics*, High Education Press, Beijing (in Chinese).

[61] Wang, Z. X., 1965, *Introduction to Statistical Physics*, High Education Press, Beijing (in Chinese).

[62] Landau, L. D., and Lifshitz, E. M., 1958, *Statistical Physics*, translated by E. Peierls and R. F. Peierls, Pergamon Press, London.

[63] Jiang, D. Q., Qian, M., and Qian, M. P., 2004, *Mathematical Theory of Non-equilibrium Steady State*, Springer, Berlin.

[64] Kadanoff, L. P., 2000, *Statistical Physics (Statics, Dynamics and Renormalization)*, World Scientific, Singapore.

[65] Grüneisen, E., 1926, *Handbuch der Physik*, Springer, Berlin (in German).

[66] Aidun, J. B., Lo, D. C. S., Trucano, T. G., and Fye, R. M., 1999, "Representative Volume Size: A Comparison of Statistical Continuum Mechanics and Statistical Physics," SAND99-1176.

[67] Xia, M. F., Song, Z. Q., Xu, J. B., Zhao, K. H., and Bai, Y. L., 1996, "Sample-Specific Behavior in Failure Models of Disordered Media," Commun. Theor. Phys., **25**(1), pp. 49–54.

[68] Arcangelis, L. D., 1989, "Scaling Behavior in Fracture Models," Phys. Scr., T, **T29**, pp. 234–238.

[69] He, G. W., Xia, M. F., Ke, F. J., and Bai, Y. L., 2004, "Multiscale Coupling: Challenges and Opportunities," Progress in Natural Sciences, **14**, pp. 463–466.

[70] Wang, H. Y., He, G. W., Xia, M. F., Ke, F. J., and Bai, Y. L., 2004, "Multi-Scale Coupling in Complex Mechanical Systems," Chem. Eng. Sci., **59**, pp. 1677–1686.

[71] Bai, Y. L., Xia, M. F., Wei, Y. J., and Ke, F. J., 2002, "Non-Equilibrium Evolution of Collective Microdamage and its Coupling With Mesoscopic Heterogeneities and Stress Fluctuations," *High-Pressure Shock Compression of Solids VI*, Y. Horie, L. Davison, N. N. Thadhani, eds., Springer, NY, pp. 255–278.

[72] Barenblatt, G. I., 1992, "Micromechanics of Fracture," *Theoretical and Applied Mechanics*, S. R. Bodner, J. Singer, A. Solan, and J. Hashin, eds., Elesevier Science, Amsterdam, pp. 25–52.

[73] Orowen, E., 1934, "Zur kristallplastizität," Z. Phys., **89**, pp. 605–659, (in German).

[74] Taylor, G., 1934, "Mechanism of Plastic Deformation of Crystals, Part I: Theoretical," Proc. R. Soc. London, Ser. A, **A145**(855), pp. 362–387.

[75] Polyani, M., 1934, "Über Eine Art Gitterströrung, die Einen Kristall Plastisch Machen Könnte," Z. Phys., **89**, pp. 660–664, (in German).

[76] Kroener, E., 1958, "Kontinuumstheorie der Versetzungen und Eigenspannungen," *Handbook of Physics*, Springer, Berlin.

[77] Kroener, E., 1962, "Dislocation and Continuum Mechanics," Appl. Mech. Rev., **15**(8), pp. 599–606.

[78] Nix, W. D., and Gao, H., 1998, "Indentation Size Effects in Crystalline Materials: A Law for Strain Gradeinr Plasticity," J. Mech. Phys. Solids, **46**(3), pp. 411–425.

[79] Dai, L. H., Ling, Z., and Bai, Y. L., 1999, "A Strain Gradient-Strengthening Law for Particle Reinforced Metal Matrix Omposites," Scr. Mater., **41**(3), pp. 245–251.

[80] Sih, G. C., 2000, "Micromechanics Associated With Thermal/Mechanical Interaction for Polycrystals," *Mesomechanics 2000*, G. C. Sih, ed., Tsinghua University Press, Beijing, pp. 1–18.

[81] Panin, V. E., 1998, "Overview on Mesomechanics of Plastic Deformation and Fracture of Solids," Theor. Appl. Fract. Mech., **30**(1), pp. 1–11.

[82] Golebiewska-Lasota, A. A., 1979, "Dislocation and Gauge Invariance," Int. J. Eng. Sci., **17**(3), pp. 329–333.

[83] Golebiewska-Lasota, A. A., and Edelen, D. G. B., 1979, "On the Gauge Transformations Admitted by the Equations of Defect Dynamics," Int. J. Eng. Sci., **17**(3), pp. 335–339.

[84] Kadic, A., and Edelen, D. G. B., 1983, "A Gauge Theory of Dislocation and Disclinations," *Lecture Notes in Physics*, Springer-Verlag, Heidelberg.

[85] Duan, Z. P., Huang, Y. L., and Wang, W. B., 1989, "Gauge Field Theory of Defects," Adv. Mech., **19**(2), pp. 172–194 (in Chinese).

[86] Weibull, W., 1951, "A Statistxcal Distribution Function of Wide Applicability," ASME J. Appl. Mech., **18**(3), pp. 293–297.

[87] Coleman, B. D., 1958, "On the Strength of Classical Fibers and Fiber Bundles," J. Mech. Phys. Solids, **7**(1), pp. 60–70.

[88] Daniel, H. E., 1945, "The Statistical Theory of the Strength of Bundles," Proc. R. Soc. London, Ser. A, **A183**(995), pp. 405–435.

[89] Harlow, D. G., and Phoenix, S. L., 1978, "The Chain-of-Bundles Probability Model for the Strength of Fiberous Materials, I: Analysis and Conjectures," J. Compos. Mater., **12**(2), pp. 195–214.

[90] Du, S. Y., and Wang, B., 1998, *Mesomechanics of Composite Materials*, Science Press, Beijing, (in Chinese).

[91] Krajinovic, D., 1996, *Damage Mechanics*, Elsevier Science B.V., Amstredam.

[92] Xia, M. F., Zhang, X. H., Xu, X. H., Ke, F. J., and Bai, Y. L., 2002, "Catastrophe Transition and Critical Sensitivity in Brittle Media With Mesoscopic Heterogeneity," *Int Symp. of Multiscaling in Mechanics*, G. C. Sih and C. P. Spyropoulos, eds., National Technical University of Athens Press, Athens, pp. 96–101.

[93] Curran, D. R., Seaman, L., and Shockey, D. A., 1987, "Dynamic Failure of Solids," Phys. Rep., **147**(5–6), pp. 253–388.

[94] Cock, A. C. F., and Ashby, M. F., 1982, "On Creep Fracture by Void Growth," Prog. Mater. Sci., **27**(3–4), pp. 189–244.

[95] McClintock, F. A., 1974, "Statistics of Brittle Fracture," *Fracture Mechanics of Ceramics*, R. C. Bradt, ed., Plenum Press, New York, pp. 93–114.

[96] Xing, X. S., 1966, "Statistical Theory of Brittle Fracture," Acta Phys. Sin., **22**(4), pp. 487–493 (in Chinese).

[97] Xing, X. S., 1986, "The Microscopic Mechanism and Non-Equilibrium Statistical Nature of Brittle Fracture," Adv. Mech., **16**(4), pp. 495–510 (in Chinese).

[98] Bai, Y. L., Ke, F. J., and Xia, M. F., 1991, "Formulation of Statistical Evolution of Microcracks in Solids," Acta Mech. Sin., **7**(1), pp. 59–66.

[99] Xia, M. F., Han, W. S., Ke, F. J., and Bai, Y. L., 1995, "Statistical Meso-Scopic Damage Mechanics and Damage Evolution Induced Catastrophe," Adv. Mech., **25**(1), pp. 1–40, 145–173 (in Chinese).

[100] Bai, Y. L., Ling, Z., Luo, L. M., and Ke, F. J., 1992, "Initial Development of Microdamage Under Impact Loading," ASME J. Appl. Mech., **59**(3), pp. 622–627.

[101] Han, W. S., Xia, M. F., Shen, L. T., and Bai, Y. L., 1997, "Statistical Formulation and Experimental Determination of Growth Rate of Micrometre Cracks Under Impact Loading," Int. J. Solids Struct., **34**(22), pp. 2905–2925.

[102] Ke, F. J., Bai, Y. L., and Xia, M. F., 1990, "Evolution of Ideal Micro-Crack System," Sci. China, Ser. A: Math., Phys., Astron. Technol. Sci., **33**(12), pp. 1447–1459.

[103] Han, W. S., and Bai, Y. L., 1995, "Embryo-Damage Induced Nucleation of Microcracks in an Aluminium Alloy Under Impact Loading," Acta Metall. Mater., **43**(6), pp. 2157–2162.

[104] Xia, M. F., Ke, F. J., Lu, Y. H., and Bai, Y. L., 1991, "Effect of Stochastic Extension in Ideal Microcrack System," Sci. China, Ser. A: Math., Phys., Astron. Technol. Sci., **34**(5), pp. 579–589.

[105] Li, H. L., and Huang, Z. P., 1996, "Effect of Evolution Rate-Sensitivity on Statistical Evolution of Microvoids," Sci. China, Ser. A: Math., Phys., Astron., **39**(12), pp. 1332–1339.

[106] Li, Q., He, Z. R., and Song, M. S., 1996, "Nonequilibrium Statistical Theory of Damage and Fracture for Glassy Polymers, 1: The Statistical Distribution and Evolution of Microcracks in Glassy Polymers," Macromol. Theory Simul., **5**(2), pp. 183–197.

[107] Hong, Y. S., Lu, Y. H., Fang, B., and Bai, Y. L., 1997, "Collective Evolution Characteristics and Computer Simulation of Short Fatigue Cracks," Philos. Mag. A, **75**(6), pp. 1517–1531.

[108] Van Papenfuss, P. C., and Muschik, W., 2003, "Mesoscopic Theory of Microcracks," Arch. Mech., **55**(5–6), pp. 481–499.

[109] Papenfuss, C., Van, P., and Muschik, W., 2000, "Mesoscopic Dynamics of Microcracks," Phys. Rev. E, **62**(5), pp. 6206–6215.

[110] Papenfuss, C., Van, P., and Muschik, W., 2002, "Griffith Cracks in the Mesoscopic Microcrack Theory," published online, Condensed Matter, abstract, cond-mat/0211207.

[111] Lemanska, M., Englman, R., and Jaeger, Z., 1997, "Transport Treatment of Crack Population in Finite Medium," Int. J. Impact Eng., **19**(3), pp. 257–272.

[112] Kiselev, S. P., and Kiselev, V. P., 2002, "Superdeep Penetration of Particles into a Metal Target," Int. J. Impact Eng., **27**(2), pp. 135–152.

[113] Bai, Y. L., Bai, J., Li, H. L., Ke, F. J., and Xia, M. F., 2000, "Damage Evolution, Localization and Failure of Solids Subjected to Impact Loading," Int. J. Impact Eng., **24**(6–7), pp. 685–701.

[114] Bai, Y. L., Xia, M. F., Ke, F. J., and Li, H. L., 2002, "Closed Trans-Scale Statistical Microdamage Mechanics," Acta Mech. Sin., **18**(1), pp. 1–17.

[115] Wang, H. Y., Bai, Y. L., and Wei, Y. J., 2002 "Analysis of Spallation Based on Trans-Scale Formulation of Damage Evolution," *Proc. of 2nd Int. Conf. on Structural Stability and Dynamics*, C. M. Wang, G. R. Liu, and K. K. Ang, eds., World Scientific, Singapore, pp. 955–964.

[116] Wang, H. Y., Bai, Y. L., Xia, M. F., and Ke, F. J., 2004, "μm- and μs-Damage Evolution, Energy Dissipation and its Trans-Scale Effects on Macroscopic Failure," *Mechanics of Materials* (in press).

[117] Bai, Y. L., Han, W. S., and Bai, J., 1997, "A Statistical Evolution Equation of Microdamage and its Application," ASTM STP 1315, pp. 150–162.

[118] Holland, J. H., 1998, *Emergence: From Chaos to Order*, Addison-Wesley, Redwood City, CA.

[119] Economou, D. J., Panagopoulos, L., and Meyyappan, M., 1998, "Examining Scale-up and Computer Simulation in Tool Design for 300-mm Wafer Processing," Microelectronics, **16**(7), pp. 101–113.

[120] Bai, Y. L., and Dodd, B., 1992, *Adiabatic Shear Localization*, Pergamon Press, Oxford.

[121] Nicolis, G., and Prigogine, I., 1977, "Self-Organization in Nonequilibrium Systems," Wiley, New York.

[122] Bai, Y. L., Xia, M. F., Wang, H. Y., and Ke, F. J., 2003, "Characteristic Dimensionless Numbers in Multi-Scale and Rate-Dependent Processes," China Particuology, **1**(1), pp. 7–12.

[123] Xia, M. F., Ke, F. J., Wei, Y. J., Bai, J., Bai, Y. L., 2000, "Evolution Induced Catastrophe in a Nonlinear Dynamical Model of Material Failures," Nonlinear Dyn., **22**(2), pp. 203–224.

[124] Xia, M. F., Ke, F. J., Bai, J., Bai, Y. L., 1997, "Threshold Diversity and Trans-Scales Sensitivity in a Nonlinear Evolution Model," Phys. Lett. A, **236**(1–2), pp. 60–64.

[125] Xia, M. F., Wei, Y. J., Ke, F. J., and Bai, Y. L., 2002, "Critical Sensitivity and Trans-Scale Fluctuations in Catastrophic Rupture," Pure Appl. Geophys., **159**(10), pp. 2491–2509.

[126] Sahimi, M., and Arbabi, S., 1993, "Mechanics of Disordered Solids," Phys. Rev. B, **47**(2), pp. 713–722.
[127] Jaeger, J. C., and Cook, N. G. W., 1979, *Fundamentals of Rock Mechanics*, Chapman & Hall, London.
[128] Zhang, X. H., Xu, X. H., Xia, M. F., Ke, F. J., and Bai, Y. L., 2004, "Critical Sensitivity in Driven Nonlinear Threshold Systems," Pure Appl. Geophys., **161**(9–10), pp. 1931–1944.
[129] Sornette, D., 2000, *Critical Phenomena in Natural Sciences*, Springer-Verlag, Berlin.
[130] Rundle, J. B., Klein, W., Turcotte, D. L., and Malamud, D., 2000, "Precursory Seismic Activation and Critical Point Phenomena," Pure Appl. Geophys., **157**(11–12), pp. 2165–2182.

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