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Evolution induced catastrophe

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Abstract

Fracture due to coalescence of microcracks seems to be catalogued in a new model of evolution induced catastrophe (EIC). The key underlying mechanism of the EIC is its automatically enlarging interaction of microcracks. This leads to an explosively evolving catastrophe. Most importantly, the EIC presents a fractal dimension spectrum which appears to be dependent on the interaction.

Fracture of materials is a catastrophic event. So far, it has been widely accepted that fractured surfaces can be modeled by fractals. However, it has also been noticed that a mere fractal description of a fractured surface can hardly benefit our understanding of the mechanism underlying this type of catastrophe [1]. Therefore, in recent years, a number of works have attempted to reveal the transition from gradual accumulation of microdamage to eventual fracture, in accord with some new concepts of statistical physics, such as fractals, multifractals, scaling laws, percolation, etc. [2,3]. Although some important advances have been made, further research is obviously necessary because fracture is not only related to the disorder of materials but also to the complex physical processes of damage evolution [4].

Fracture is a non-equilibrium process by nature, and correlations between microdamage at various length scales may happen almost simultaneously. Therefore, it represents a new prototype model in nature. At first sight, this appears to be similar to the so-called self-organized criticality, such as a sandpile model [5]. In the sandpile model, sand grains are added one by one on top of the sandpile. As soon as a local slope of the pile attains a critical value, a local slide will start. This may trigger cascades, where a large catastrophic event results from a series of chain reactions which obey the same local critical condition. Hence, a large scale avalanche forms merely depending on the local slope, no matter how large the avalanche itself is. When we examine fracture owing to coalescence of microdamage and the restructuration of the stress field, something distinctly different happens.

As an illustration [6], let us examine the example of two aligned microcracks of length c a distance d apart from each other. The exact solution of the elasticity mechanics or dimensional analysis easily leads to the following expression of the stress distribution on the segment of d,

$$\frac{\sigma}{E}=f(r/d,\, c/d,\, \sigma_0/E)\;, \tag{1}$$

where σ_0 is the remote uniform tensile stress, E is the

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elastic modulus, and r is the coordinate. The average stress on the segment of d would be

$$\frac{\bar{\sigma}}{E}=\int_{-d/2}^{d/2} f(r/d, c/d, \sigma_0/E)\,\frac{\mathrm{d}r}{d}$$
$$=F(c/d, \sigma_0/E)\ . \qquad (2)$$

Suppose that the two cracks would coalesce, when the average stress attains a critical value σ_c, then the critical condition can be written as

$$F(c/d, \sigma_0/E)\geqslant \frac{\sigma_c}{E}\ , \qquad (3)$$

or

$$\frac{d}{c}\leqslant g(\sigma_0/E, \sigma_c/E)\ . \qquad (4)$$

For a specified material, material parameters such as E are fixed, hence condition (4) becomes

$$\frac{d}{c}\leqslant L_c(\sigma_0/\sigma_c)\ , \qquad (5)$$

where L_c should be an increasing function of its argument σ_0/σ_c. This condition is one of geometrical similarity. This implies that the longer the cracks, the larger their interacting regions, under the same value of σ_c/σ_0. Noticeably, this is an automatically enlarging correlation, rather than a fixed critical condition, like

$$d\leqslant d_c\ . \qquad (6)$$

As a matter of fact, expression (6) would be the counterpart of the critical local slope in the sandpile. Moreover, if the interacting range in a system is increasingly or proportionally dependent on the size of the evolving subsystem, the evolution of the system might fall in the catalogue EIC [6].

In this Letter, we shall confine ourselves to the type of geometrical similarity (5) only. The importance of the non-local nature of the critical condition (5) will become even more clear in a numerical simulation. For instance, the EIC cannot be simulated by making use of cellular automata (CA), because the non-local condition (5) will unavoidably lead to too many rules in a CA simulation, about $2^N N$. A sandpile can be easily simulated by CA. This situation leads us to work out a new algorithm and theory to explore the essence of the EIC.

We have carried out some experimental study on fracture due to coalescence of microcracks [7]. The tests were performed with a one-stage light gas gun with a 101 mm bore. This configuration of the experiment guarantees a uniform one-dimensional strain state in the sample of thickness 5–10 mm. After impact testing, the loaded sample was sectioned and the microdamage resulting from different load durations ranging from about 0.1 to 1 μs was examined. Some essential features of the microdamage are:

(1) Parallel microcracks on the cross section of the specimen are nucleated within the second-phase particles in the aluminium alloy sample, and have roughly the same size distribution as the second-phase particles.

(2) The coalescence of nucleated microcracks seems to occur suddenly.

(3) The fractured surfaces of samples show statistically self-similar network patterns within length scales of 10 μm–1 mm (the characteristic length scale of second-phase particles is about 3–4 μm), see Fig. 1. However, Fig. 1E shows a distinct fragment pattern in the range less than 10 μm. Also the correspondent fractured profiles on the sectional surfaces appear to be a fractal with dimension $D=1.09\pm0.01$ (seven sample average, scaling range 10 μm–1 mm), a very low fractal.

These experimental observations combined with the non-local condition of coalescence of microcracks triggers a conceptual model: the evolution-induced catastrophe. The main ideas and their concrete descriptions of fracture are as follows:

In a discrete two-dimensional network, parallel microcracks are nucleated randomly with a certain size distribution. Provided a crack of length c satisfies the coalescence condition, like (5), it will coalesce with its neighbor, otherwise, the crack will be locally stable. If all coalesced cracks no longer satisfy the coalescence condition, the system will be globally stable, otherwise the coalescence will continue on greater and greater scale until complete fracture. Therefore, this non-local coalescence condition together with continuing crack nucleation will lead to a cascade of coalescence of microcracks. As an additional aspect in the two-dimensional coalescence, we supposed that a uniform probability of coalescence

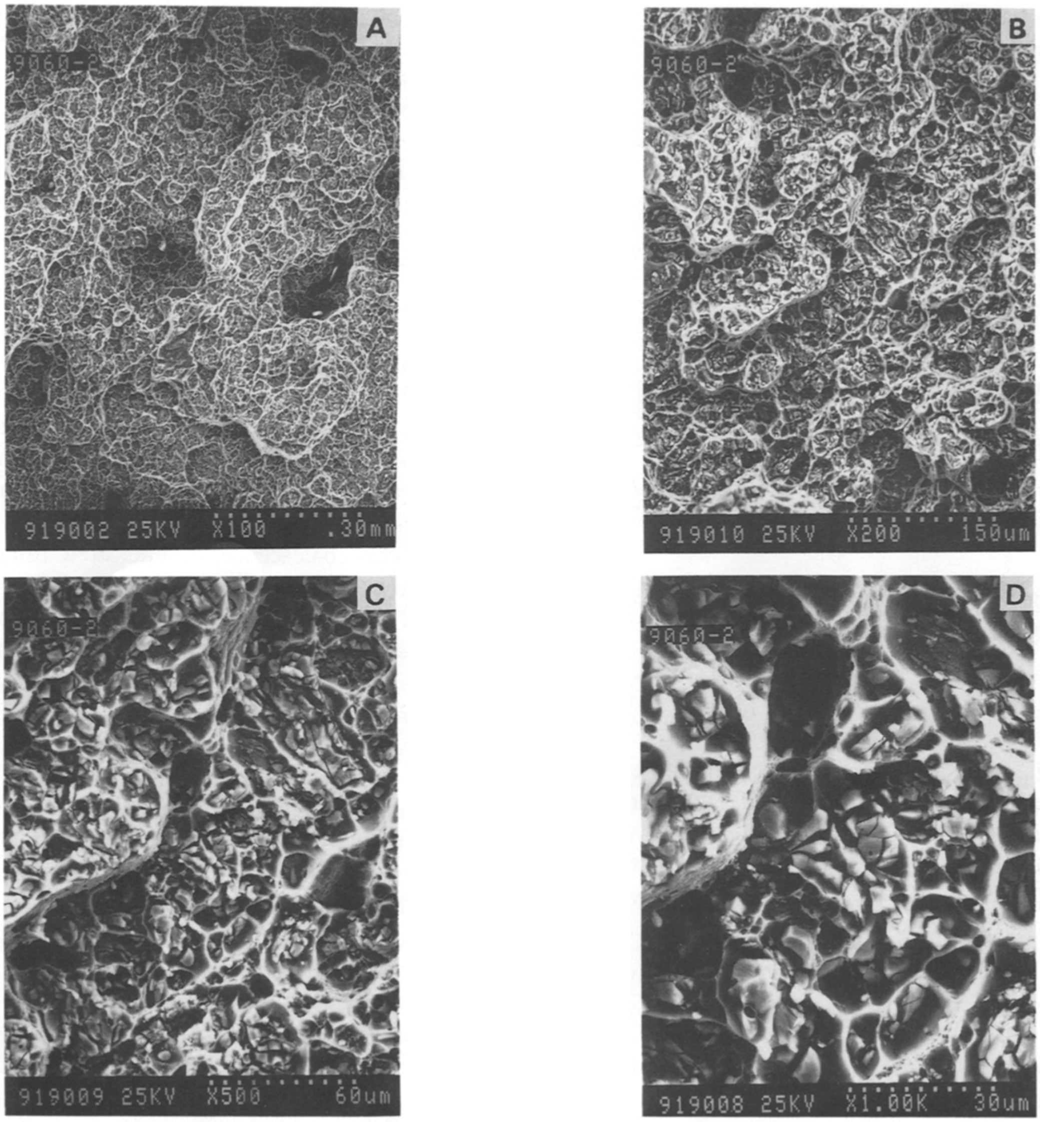

Fig. 1. Top view fractographs of the spalled surface under different magnifications.

held provided $-\frac{1}{2}\pi \leqslant \alpha \leqslant \frac{1}{2}\pi$, where α is the angle between the direction of cracks and their ligament, in accord with the observed coalescence pattern.

In a practical simulation of the crack coalescence, condition (5) is specified by the following rules:

(i)

$$\frac{d}{\bar{c}} \leqslant L_c , \tag{7}$$

where $\bar{c} = \frac{1}{2}(c_1 + c_2)$ and c_1 and c_2 are two adjacent cracks and d is the distance between two cracks.

(ii) The priority of coalescence gives the nearest crack satisfying (7), and is independent of α, if $-\frac{1}{2}\pi \leqslant \alpha \leqslant \frac{1}{2}\pi$.

(iii) The length of a curved crack (coalesced crack) in the further calculation of coalescence is represented by its projection normal to the tensile loading.

Fig. 1. Continued.

Therefore, in an EIC, a single concise rule (7) can cover all size interactions. Moreover, the algorithm in an EIC is Lagrangian, i.e. tracing each crack, instead of Eulerian in CA, i.e. tracing each site in discrete space. This two features make an EIC extremely simple in computation.

Our simulation was carried out on a $L\times L=500\times 500$ network, with mesh size $\Delta x=\Delta y=\frac{1}{2}c_0$, where c_0 is the characteristic length of the nucleated crack. For example, our experiments showed $c_0=4$ μm, then $\Delta x=\Delta y=2$ μm and the network is 1 mm^2. So the unit mesh represents half of a nucleated crack.

Several distinct features of this evolution induced catastrophe are:

(1) The catastrophic coalescence of microcracks happens explosively. Before the critical transition, a minor coalescence of microcracks happens here and there and then the system becomes globally stable, there is no hint of a catastrophe. However, suddenly a trivial newly nucleated microcrack initiates successive coalescence, more and more microcracks are involved in the event of coalescence and form the eventual fracture (Fig. 2).

(2) The critical fraction of broken bonds in an individual simulation is definite, but it changes from case to case in the range 0.20–0.28, along the trajectory of the fracture profile and 0.01–0.02 on the network. The existence of a transitional region in an EIC rather than a critical value has been justified in a one-dimensional evolution model [8].

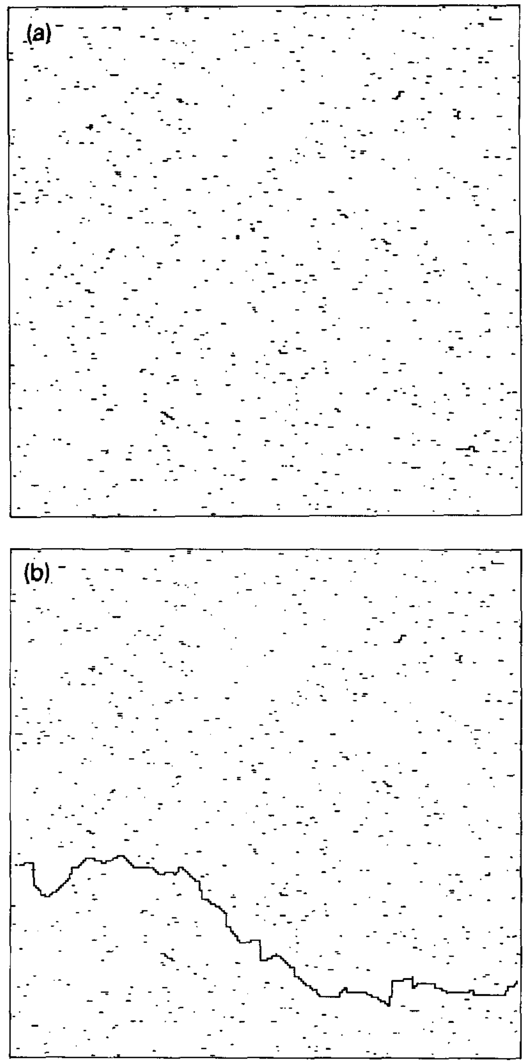

Fig. 2. Patterns of the computer simulation of the microcrack system. (a) Pattern of microcracks just before EIC. (b) Pattern of EIC triggered by a newly nucleated crack in (a).

(3) The fractal dimension of the curved fracture seems to be a constant, $D=1.09$ (ten sample average with standard deviation 0.01) when $L_c=1$. This value is in good agreement with the observed one, $D=1.09$ (compare Figs. 2b and 3). This implies that the evolution induced catastrophe is highly localized. Clearly,

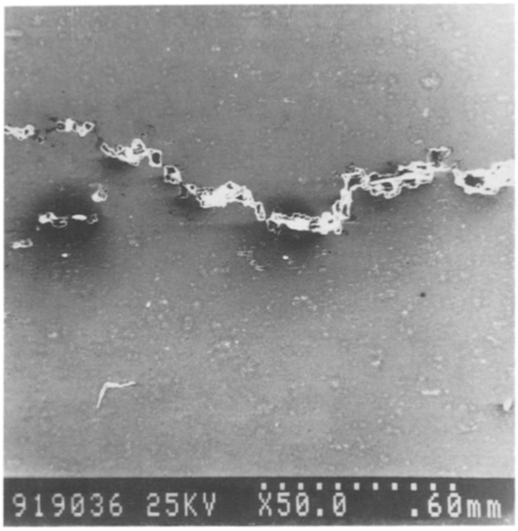

Fig. 3. Sectional picture of the spalled profile.

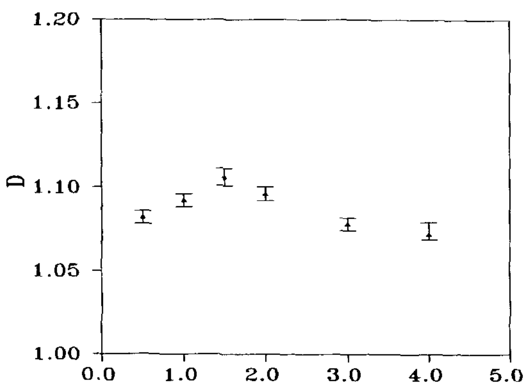

Fig. 4. Dependence of the fractal dimension on the interaction L_c of microcracks in our two-dimensional simulation.

the fractal is independent of the characteristic size of the nucleated cracks c_0, because the measurement of the fractal dimension is based upon the dimensionless stick. Furthermore, our simulation shows that the fractal dimension D is independent of the form of the size distribution too. We have tested five different size distributions of crack nucleation: Rayleigh's, power, exponential, step and triangular functions, the variation of D falls within ± 0.02. All these indicate that the EIC may constitute a representative group. However, the fractal dimension D is dependent on the interaction parameter L_c. This implies that the interaction parameter L_c governs a spectrum $D = D(L_c)$ [9].

Fig. 4 presents some of the variation of the fractal dimension with the interaction in our two-dimensional simulation. The variation could be qualitatively estimated by an over-simplified statistical approach as

$$D = \frac{\log(2 + L_c/2)}{\log(2 + L_c/\pi)} . \tag{8}$$

For $L_c = 1$, we have $D = 1.0897$ [9]. This formula also demonstrates a unimodal spectrum $D(L_c)$, but has a peak value much higher than that obtained in the simulation.

Since a number of interactions in nature may be expressed in a way similar to (5), an enlarging interaction, the EIC may represent a simple model to describe the catalogue.

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