Plugging: physical understanding and energy absorption

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Research into plugging and related processes, such as punching and blanking, is briefly reviewed and discussed with emphasis on understanding the physical aspects of the process and its mechanical description. Based on progress in the study of thermoplastic shear instability, a mechanical model is proposed which concentrates on localization of shear deformation up to the occurrence of the instability; the latter phenomenon has been observed experimentally. Making two additional assumptions about perforation and strain distribution, an overestimate of the amount of energy absorbed is obtained, which accounts for well known empirical formulae. Also, an approach is given for assessing the ability of plate materials to withstand this type of fracture. MT/816

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List of symbols

a=radius of punch
A=area
b=thickness of plate
B=non-dimensional thickness of plate (b/a)
C_V=specific heat
d=diameter of punch
e=energy absorption due mainly to material
$\bar{e}$=non-dimensional energy absorption
$\tilde{e}$=energy absorption per unit volume
E=energy absorption
f=constant
k=yield shear stress
K=material constant
l=terminating depth of penetration
m=mass of plug
M=mass of punch
n=work-hardening index
p=depth of penetration
r=radial variable
$\bar{r}$=non-dimensional radius
R=resistance of target
S=measure of strength of plate
t=time
v=velocity of penetration
W_p=plastic work done
α=thermal softening parameter
β=constant
γ=shear strain
δ=width of shear band
η=fractional non-dimensional penetration
θ=temperature
κ=kinematic viscosity
ξ=non-dimensional penetration
ρ=density
σ=stress
τ=shear stress

Subscripts

a=at periphery of punch
c=critical for perforation
f=residual
i=thermal shear stability
j=initial
M=maximum
q=non-elastic collision
s=shear
t=target
z=axial direction
0=quantity at impact
*=material property

Plugging, and similar industrial processes such as cropping, blanking, punching, and shearing, are by no means new phenomena and indeed the subject has been emphasized from different angles since 1945. In 1950 Chang and Swift[1] carried out a comprehensive study of shearing, and some of their observations are still frequently quoted. In 1967, Johnson and Slater[2] published a long survey on slow and fast blanking, and the basic concepts they put forward are also suitable for other related processes. In particular, they reflected the background to the fresh interest shown in the study of blanking and the complexity involved in various cognate processes, namely, the effects of strain rate, the speed of operation, temperature, and stress-wave propagation. In 1978 Backman and Goldsmith,[3] in their comprehensive review, surveyed historical and current research work on penetration. Plugging, of course, received much attention, and according to their review it is not hard to trace the evolution of the concepts in this process. It is also quite clear that an understanding of the real physical process and of the interrelationship between various physical parameters and different phenomenological features becomes crucially significant. In this sense, Johnson and Slater[2] and Backman and Goldsmith[3] share the same point of view, although their starting points are quite different. Their comments represent the main trend in research on plugging and related processes. From an academic point of view, this trend inevitably stimulates fresh interest in the subject. As will be pointed out, some new concepts have been successively introduced in recent years, but the total understanding of all the physical aspects involved is still far from satisfactory.

In this paper, historical progress is first outlined, and some important results are described, particularly with reference to recent publications. It is desirable in the course of this historical description to clarify useful approaches which can benefit research in both academic and technical aspects. Later, a simple model is developed mainly for identifying the physical features implied by these processes, and more specifically – based on results about thermoplastic instability obtained in the past two decades – a physical description is put forward. This model enables a number of empirical observations to be explained. With this physical understanding, it becomes possible to clarify some of the effects of the material and geometrical parameters in punch-load variation and energy absorption. Also, an overestimate of the energy absorbed can be obtained, which can provide a foundation for empirical expressions.

Brief survey

Early studies of plugging were concerned only with critical impact velocity or critical energy absorption, and this would seem to have been prompted entirely by military interests and implications.[4]

The easiest approach to impact velocity and energy absorption uses dimensional analysis. The physical quantities involved in this phenomenon are the plate thickness b, some measure S of the strength of the plate, and the radius a and mass M of a projectile. Thus, the critical perforation velocity v_c can be written as

$$v_c = v_c(a, b, M, S) \quad (1)$$

Application of the usual methods of dimensional analysis and the π-theorem reduces the above expression to

$$\frac{Mv_c^2}{Sa^3} = f\left(\frac{b}{a}\right) \quad (2)$$

Further, a parabolic approximation to the function gives

$$\frac{Mv_c^2}{a^3} = K\left(\frac{b}{a}\right)^{\beta} \quad (3)$$

where K is a material constant and β another constant.

The French engineer de Marre summarized gross data to yield the well known de Marre formula for steel plate,

$$\frac{Mv_c^2}{d^3} = K\left(\frac{b}{d}\right)^{1.43} \quad (4)$$

where d is the diameter of the projectile.[4] This equation has long been widely used in blunt-projectile-perforation studies, but even though some textbooks (*see* for instance Ref.5) have quoted this kind of formula, no sound physical explanation of its origin has been offered. For example, why do the two geometrical parameters constitute such a form, and how is the target material to be accounted for or included since it sustains this kind of failure and implies some special energy-absorption mechanism? These points require a deeper physical understanding, for which dimensional analysis is no longer satisfactory.

That all the early works are phenomenological may be justified by reference to both the problem and the demands for practical solutions. The forces responsible for plugging have not been examined in depth. Basically, there are two approaches. The first is the 'force approach', with an analysis based on the mechanical-equilibrium equation; Spells's paper[6] is an early example of this. He simply assumed the resistance R of the target to be given by

$$R = A_t \rho_t v^2 \quad (5)$$

where A_t and ρ_t are the area and density of the target, respectively, and v is the penetration velocity. Integrating the kinetic equation for the projectile led to

$$v = v_j \exp\left(-\frac{A_t \rho_t p}{M}\right) \quad (6)$$

where p is the depth of penetration and v_j the initial velocity of entry. It now becomes clear that equation (6) might be applicable to some kinds of penetration, but not for plugging.

The second approach is the 'energy approach', used mainly by Recht and Ipson,[7] and can be outlined as follows. The total kinetic energy of the projectile $Mv_0^2/2$ is dissipated in three ways: as residual kinetic energy $(M+m)v_f^2/2$ (where v_f denotes the residual velocity and m the mass of the plug); as shear deformation energy E_s, and as nonelastic collision energy absorption $E_q = Mmv_0^2/2(M+m)$. Because the relationship between E_s and the material properties was not specified, the whole problem of energy partition remains unsolved.

It has been realized gradually that neither oversimplified material resistance nor confusing assumptions about energy absorption can lead to satisfactory results, and thus that the plugging process itself must be examined. In the mid-1960s Pytel and Davids[8] and Minich[9] used viscoplastic theory to describe the process and carried out some numerical simulations. Their models embody the following equations:

$$\frac{\partial \sigma_{rz}}{\partial r} + \frac{\sigma_{rz}}{r} = \rho_t \frac{\partial v}{\partial t} \quad (7)$$

$$(m+M)\frac{\partial v}{\partial t} = 2\pi ab\left(-k + \kappa\frac{\partial v}{\partial r}\right), \quad r = a \quad (8)$$

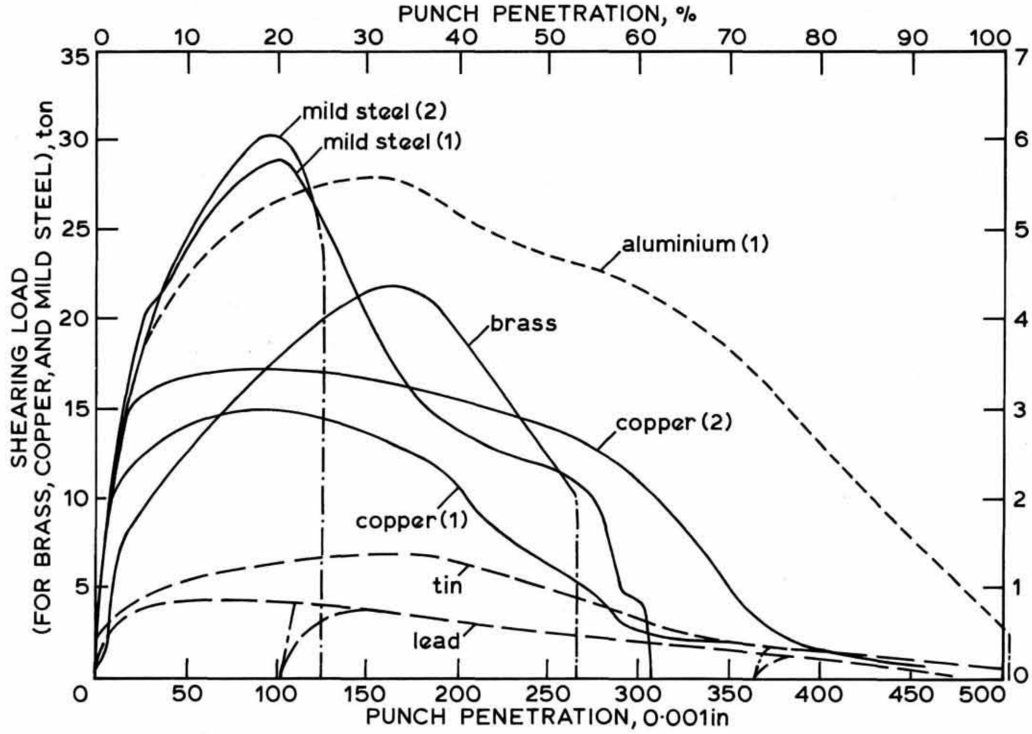
$$v(r, 0) = 0, \qquad r > a \quad (9)$$

$$v(r, 0) = v_j, \qquad r \le a \quad (10)$$

where κ denotes kinematic viscosity and k the yield shear stress. Their simulation provided a much better understanding than was hitherto available.

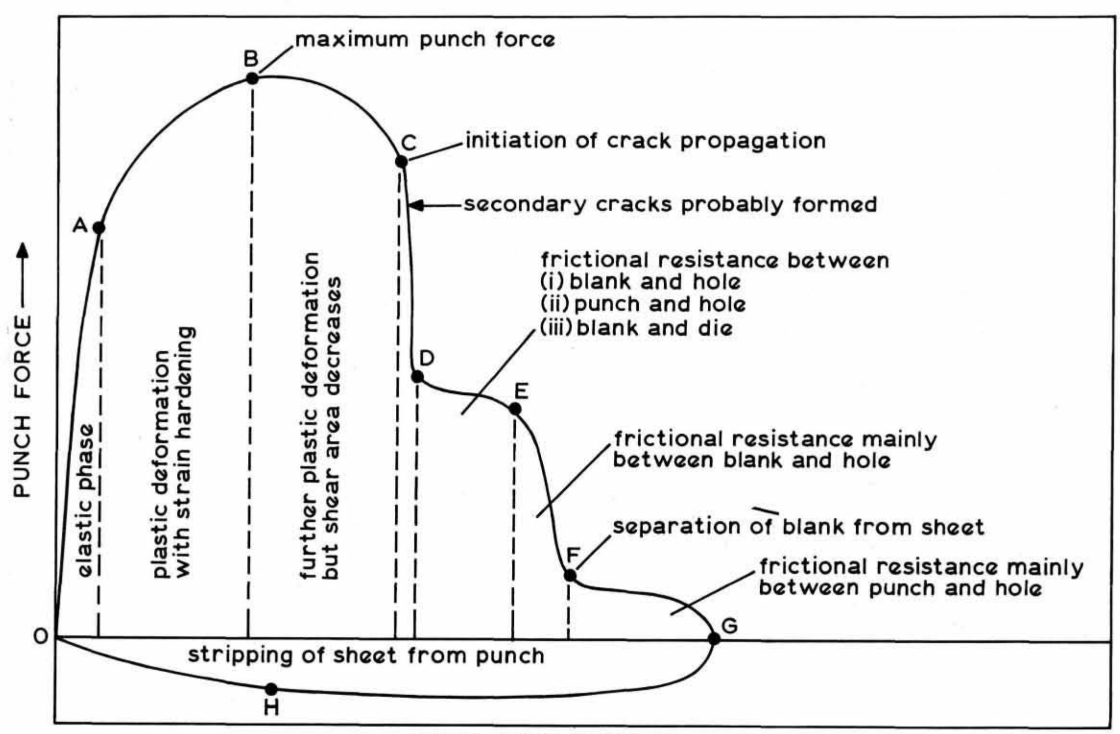
During this period another development occurred which took account of projectile deformation and various kinds of deformation in the target. Awerbuch and Bodner[10] and Marom and Bodner[11] aimed to develop a model which included the latter effects, and consisted of three interconnected stages. The first occurred mainly during the indentation of the target by a deformable punch. This was followed by compression and shear deformation. Finally, the plug and projectile were considered to move with the same velocity until the plug was ejected. This is a more sophisticated model than those of Pytel and Davids[8] and Minich,[9] which considered only simple shear, and were neither able to include early indentation nor suitable for thin plates. Nevertheless, the results obtained by Awerbuch and Bodner[10] showed the first stage to be rather less important than the second, which was dominated by shear deformation. The significant point in the analysis of Awerbuch and Bodner is that it includes radial clearance in relation to the width of the shear band. Most workers have taken shear force to be the key to an understanding of plugging, but little has been done to establish just how shear deformation proceeds in plugging. Both Pytel and Davids's simple shear, visco-plastic model[8] and Awerbuch and Bodner's radial clearance assumption[10] are valuable attempts in this latter direction.

It is instructive to reconsider Chang and Swift's[1] investigation. The punch load–penetration autographic diagrams, which they obtained for various metals, are as shown in Fig.1. For all the metals they examined, the curves attain a peak load after which they drop abruptly or smoothly according to the material. Obviously, this is a significant indication of behaviour in processes such as plugging and



1 Punch load–penetration autographic diagrams for various metals; after Chang and Swift[1] (1in=25·4mm; 1 ton≈0·984 tonne)

blanking. Later, Johnson and Slater,[2] in their general survey of slow and fast blanking of metals at ambient and high temperatures, outlined the individual phases shown in Fig.2, and qualitatively explained Chang and Swift's earlier experimental observations. Plastic deformation develops between A and C in Fig.2, and at B the punch force attains its maximum, which implies a kind of instability phenomenon. During the post-instability phase, plastic flow takes place between B and C, and microcracks may be initiated somewhere within the initially crack-free body, for instance at the site of imperfections. Beyond C the punch force drops dramatically, there being a coalescence of small cracks and



2 Schematic representation of punch force–displacement autographic diagram; after Johnson and Slater[2]

subsequently a macrocrack propagation. Finally, it is suggested that some frictional effects give rise to a low level of resistance (*see* the punch load–penetration diagrams in Ref.2). While this general picture is sound, there is much room for improvement, especially in respect of quantitative treatment.

Johnson and Slater pointed out at the beginning of their review that 'fresh interest in the study of the axisymmetric blanking process has been stimulated mainly by the advent of H.E.R.F. methods during the last decade' and 'the scientific importance of H.E.R.F. has been to compel, both jointly and severally, attention to the effects of strain rate and speed of operation, temperature and stress wave propagation in metals'.[2] Further, since plugging and related processes have a simple geometrical configuration, it is relatively easy to study the two fundamental aspects of ductile fracture, i.e. shear instability and the initiation and coalescence of cracks. Most significant and complicated to study are the interaction and coupling of the latter.

Parallelling the above-mentioned studies is that of thermoplastic shear instability. To some extent, research on this subject is more necessary than the development of simplified models for understanding plugging and related processes, and indeed there have been some important achievements in the past two decades. Although Backman and co-workers have performed and reported some damage-mechanism research (*see* for instance Ref.12), it is unfortunate that the comprehensive review (Ref.3) does not mention shear instability. Further, some investigations have noted and proposed the incorporation of thermoplastic shear instability into the mechanical model. Recht[13] and Recht and Ipson[7] provided clear pictures showing white adiabatic shear lines at the periphery of punch and plug. Balendra and Travis[14] (based on their dynamic blanking tests on steel of various hardnesses) suggested that, at all hardnesses up to that of the ductile–brittle transition, separation occurs in all dynamic tests (often with varying amounts of plastic shear) following the development of thermoplastic instability, and in no case is separation due to the mechanism effective in static tests, that is after varying amounts of plastic shear followed by fracture. Later, Zaid and Travis[15,16] found that thermoplastic instability occurs in high-speed impact with single and multi-plate targets as well. Notably, Balendra and Travis[14] and Johnson[17] stressed the distinction between the 'metallurgical separation' of blank and stock and 'blanking', i.e. physical separation in the conventional sense. Only the former, of course, is identical with adiabatic shear instability. It seems that a consistent physical description of plugging is still needed.

In carbon steel plate, a plug is usually ejected with a white etched band which separates the plug from the target. It has been held that a shear strain of about 100 occurs in the white band. In other metal plates, for instance aluminium alloy, similar intensive shear bands have also been observed. Rogers[18,19] described the process as follows: intense shear bands form in the plate at the corner of the punch; they penetrate right through to the back surface when the plate is approximately one-third penetrated[17]; cracks initiated in the shear bands on the rear of the plate progress back along the shear bands to cause final plug detachment as penetration continues.

As far as the material is concerned, Irwin[20] has found that tungsten is the most resistant to shear-band deformation, while uranium alloys are most susceptible to it, along with tungsten carbide–cobalt alloys. Culver,[21,22] using dynamic torsion tests, has measured critical adiabatic shear strains in various metals and alloys such as mild steel, stainless steel, copper, aluminium alloys, titanium, and titanium alloys. He concluded that, of these metals and alloys, titanium and its alloy Ti–6Al–4V are the most susceptible to adiabatic shear instability, whereas copper is the least susceptible.

Rogers[19] also reviewed shear instability in high-velocity forming and in chip formation in machining. From all observations, it has been concluded that the critical shear strain, and to some extent the energy absorption in plugging, is closely connected with thermal softening and strain-rate hardening of the material concerned. From this point of view, the viscostatic plastic model material seems to represent an oversimplification in that it does not include the key feature of material behaviour during the plugging process.

Recently, Atkins[23] introduced a kind of critical-instability shear strain into an examination of cropping. He assumed a constant width of shear band, but did not include any thermal effects. Obviously, a more embracing and physically reasonable description is desirable.

In this paper it is not the aim to go too far into materials science, but rather to examine some materials aspects and to draw some helpful conclusions so as to construct a physically reasonable mechanical and quantitative model for plugging and related processes.

Process of plugging

The present paper is devoted to developing a simple mechanical model which can incorporate several of the physical aspects involved in plugging, so as to provide a method of recognizing the important material properties and geometrical parameters involved. These consist of three elements. The first is the kinetic-energy equation connecting the punch or projectile and the plug, the second is the constitutive relationship, and the third is the relationship between the displacement of the plug and the shear strain. In previous papers (such as Ref.3), the first item was well discussed. As for the last two items, it seems that no one has provided a consistent and reasonable description. Some investigators have realized, however, that, apart from a characteristic strength involved in the phenomenon, critical shear strain plays a significant role in plugging. As mentioned above, the usual approach is to deal with them by assuming a characteristic strength and connecting the shear strain with the displacement of the punch using

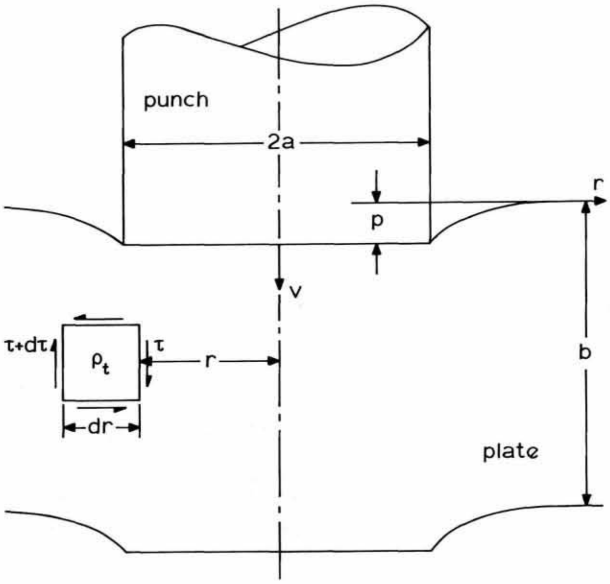
$$\gamma = p/\delta \qquad (11)$$

where δ is the width of the shear band, γ the shear strain, and p the penetration. Because there is no exact method of calculating the width, some arbitrary assumption has to be introduced.

In the present paper it is aimed to provide a description of plugging based on some fundamental assumptions concerning the constitutive relation, and relying mainly on recent progress describing adiabatic shear deformation.

It is assumed that in a punching process only simple shear deformation occurs along the direction of the moving punch when in the plate. This is, of course, suitable only where the ratio of punch diameter to plate thickness is about unity. By considering the predominant failure mode in such circumstances, the supposition, which has hitherto been commonly used, is reasonably satisfactory. Hence the equation of motion has the very simple form

$$(M+m)\frac{\mathrm{d}v}{\mathrm{d}t} = -2\pi ab\tau_{\mathrm{a}} \qquad (12)$$



3 Schematic sectional view of plugging showing parameters involved

where m denotes the mass of the plug, which is $\pi a^2 b \rho_t$; τ_a is the uniform shear stress along a cylindrical surface defined by the punch periphery at $r=a$, but inside the plate (*see* Fig.3). The velocity of penetration v, namely the velocity of the punch and the plug, is

$$v=\frac{dp}{dt} \qquad (13)$$

where p is the depth of penetration. Combining equations (12) and (13) gives

$$\frac{(M+m)}{2}\frac{d(v^2)}{dp}=-2\pi ab\tau_a \qquad (14)$$

The integration of equation (14) yields, as a measure of the absorbed energy,

$$e=\frac{(M+m)}{2}(v_j^2-v_f^2)=\int_0^l 2\pi ab\tau_a\,dp \qquad (15)$$

where v_j and v_f are the initial and final velocities of the punch and plug, and l is the ultimate penetration beyond which no energy is consumed on further penetration.

At the moment of impact, supposing that the plate remains at rest, except for the plug (and that sound and friction effects can be neglected), conservation of momentum gives

$$Mv_0=(M+m)v_j \qquad (16)$$

where v_0 is the impact velocity. Strictly speaking, the absorbed energy is

$$E=\frac{Mv_0^2}{2}-\frac{M+m}{2}v_f^2$$
$$=e+K_0\frac{m}{M+m} \qquad (17)$$

where $K_0=Mv_0^2/2$ is the initial kinetic energy of the punch. If $m \ll M$, then $E \simeq e$. It is evident that the measure of absorbed energy e, which is determined by the combination of material properties and geometrical configuration, is of crucial importance. Later, close attention will be paid to the calculation of e. For the critical impact velocity v_{0c}, which implies a zero limiting residual velocity v_f, we have

$$v_{0c}=[(M+m)2e/M]^{1/2}$$

If $m \ll M$, this reduces to

$$v_{0c}\simeq(2e/M)^{1/2} \qquad (19)$$

To discuss the plugging process further, use is made of a constitutive relation which includes temperature as well as strain; such a requirement is prescribed because of the severe localized shear deformation which arises. In Ref.17, various constitutive relations obtained at high strain rates, and including the effects of temperature and strain, are summarized. Using Bell's formula,[17] the constitutive relation may be expressed by

$$\tau=\tau_*(1+\alpha\theta)\gamma^n \qquad (20)$$

where τ_*, α, and n are material constants, and θ is the temperature. Below, only high-speed phenomena are considered, so equation (20) is adopted as an expression applicable for high strain rates, and such that strain rates need not appear explicitly in the constitutive relation.

Because of the simple shear deformation of the present case, the first law of thermodynamics for the supposedly adiabatic conditions is

$$\rho_t C_V\,d\theta=k\,dW_p$$
$$=k\tau\,d\gamma \qquad (21)$$

where C_V is the specific heat of the plate material. Here k is about 0·9 according to Taylor and Quinney,[24] but for simplicity, it will be taken here as unity.

Substituting equation (20) into equation (21) and integrating leads to

$$\tau=\tau_*\gamma^n(1+\alpha\theta_0)\exp\left(\frac{\alpha\tau_*}{\rho_t C_v}\frac{\gamma^{n+1}}{n+1}\right) \qquad (22)$$

where θ_0 is the room temperature.

As mentioned in the previous section, thermoplastic shear instability may arise because of the competition between thermal softening and strain hardening, and can be taken to occur at some critical shear strain; this can be determined[13] as

$$-\frac{\tau\left(\frac{\partial\tau}{\partial\theta}\right)}{\rho C_V\left(\frac{\partial\tau}{\partial\gamma}\right)}\geqslant 1 \qquad (23)$$

This inequality yields the critical shear strain, after using equation (22), as

$$\gamma_i=\left(-n\frac{\rho C_V}{\alpha\tau_*}\right)^{1/(1+n)} \qquad (24)$$

corresponding to which the shear stress approaches its maximum as

$$\tau_M=\tau_*\gamma_i^n(1+\alpha\theta_0)\exp[-n/(1+n)] \qquad (25)$$

Using the parameters n, γ_i, and τ_M, the adiabatic stress–strain relation (22) can be expressed in a more compact form as

$$\tau=\tau_M\left(\frac{\gamma}{\gamma_i}\right)^n \exp\left\{\frac{n}{1+n}\left[1-\left(\frac{\gamma}{\gamma_i}\right)^{n+1}\right]\right\} \quad \ldots\ldots\ldots\ldots (26)$$

It will be shown that, with the aid of the three material parameters n, γ_i, and τ_M, energy absorption in plugging can be determined more exactly.

Because the constitutive relation (20) can be regarded as defining a family of isothermal relations, from which the critical shear strain instability and the adiabatic stress–strain relations (22) and (26) are deducible, the adiabatic relation should be appropriate for both pre- and post-instability. Some typical adiabatic stress–strain curves following equation (26) are shown in Fig.4.

As a typical analysis of the penetration equation such as equation (14) shows, the absorbed energy and critical impact velocity are expressed as an integral of stress taken over the depth of penetration (equation (15)). There is no doubt that the relationship between penetration depth and shear strain is vitally important, but unfortunately it involves a complicated boundary problem, even for the case of simple shear deformation. The deformation of the plate is governed by a partial differential equation with a special boundary condition, namely

$$\frac{\partial(\tau r)}{\partial r}=r\rho_t\frac{\partial^2 p}{\partial t^2} \quad \ldots\ldots\ldots\ldots (27)$$

where at

$$r=a \quad \ldots\ldots\ldots\ldots (28)$$

p is the displacement of a plate element (*see* Fig.3). The plugging problem has to be solved by the simultaneous integration of equations (12), (13), (26), (27), and (28), and using

$$\gamma=\left|\frac{\partial p}{\partial r}\right| \quad \ldots\ldots\ldots\ldots (29)$$

In general, a solution can be effected only by numerical methods, so some researchers have adopted certain simplifying assumptions instead of using equations (27) and (29). For instance, Atkins[23] supposed that

$$\gamma_{ai}=fb \quad \ldots\ldots\ldots\ldots (30)$$

where γ_{ai} is the critical shear strain in the shear band located at the periphery of the punch, and f is a constant. But this approach is not entirely satisfactory.

Here, based on the localized shear deformation and movement, it is assumed that the deformation of the plate beyond the plug is quasi-static. Under this supposition, equation (27) is reduced to

$$\frac{\partial(\tau r)}{\partial r}=0 \quad \ldots\ldots\ldots\ldots (31)$$

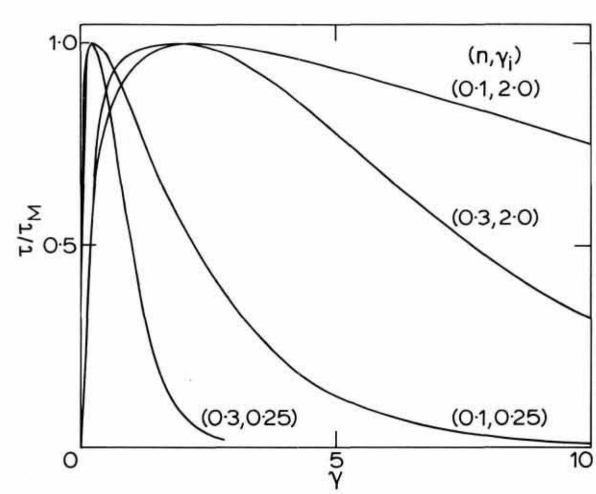
Integrating equation (31) from $r=\infty$ to $r=a$ gives the distribution of shear stress as

$$\tau=\frac{\tau_a a}{r} \quad \ldots\ldots\ldots\ldots (32)$$

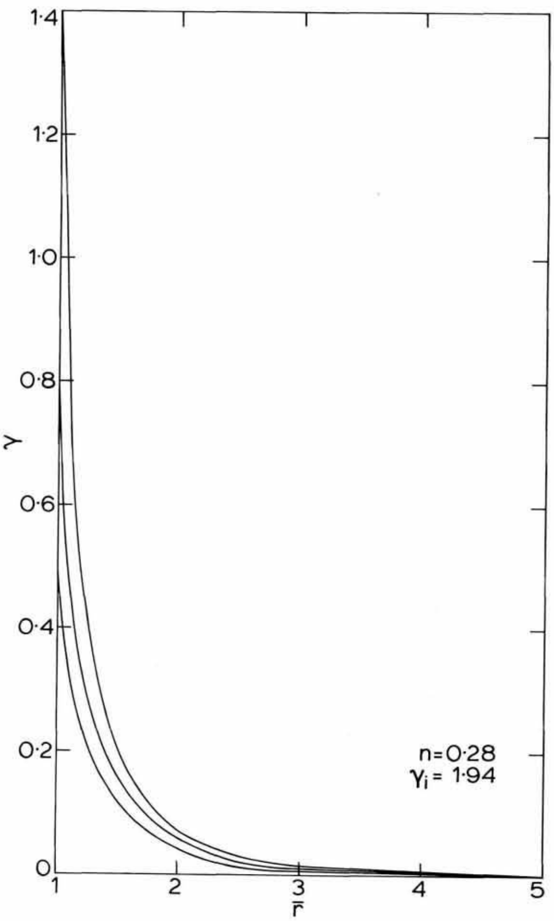
For the displacement of the plate element, equation (29) leads to

$$p=-\int_{\infty}^{a}\gamma\,dr \quad \ldots\ldots\ldots\ldots (33)$$

To integrate equation (33), an expression for the shear



4 Dimensionless adiabatic shear stress–strain diagrams for materials with various work-hardening indices *n* and critical adiabatic shear strains γ_i, after equation (26)



5 Shear-strain distribution along dimensionless radius, $\bar{r}=r/a$, for mild steel, for different shear strains at punch periphery, γ_a

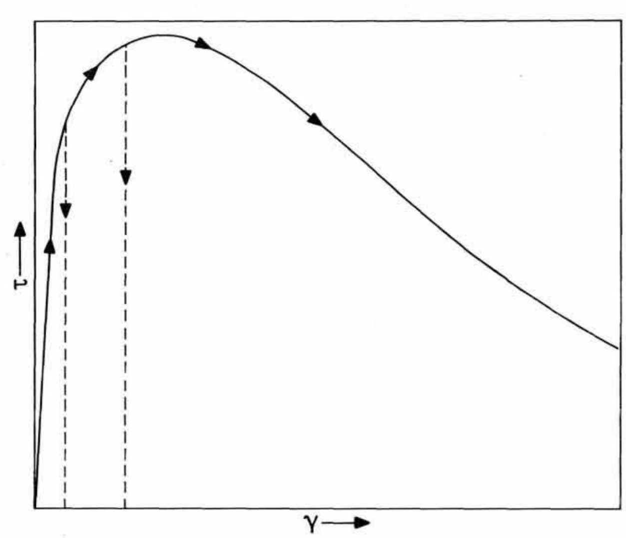
strain γ as a function of distance r is needed. Combining equations (26) and (32) yields

$$\left(\frac{\gamma}{\gamma_a}\right)=\left(\frac{a}{r}\right)^{1/n}\exp\left\{\frac{-1}{n+1}\left[\left(\frac{\gamma_a}{\gamma_i}\right)^{n+1}-\left(\frac{\gamma}{\gamma_i}\right)^{n+1}\right]\right\}\quad\ldots\ldots(34)$$

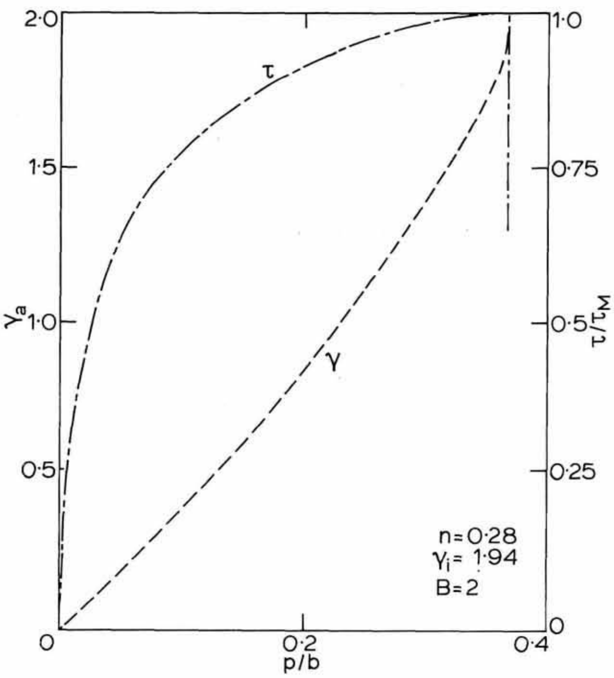
Equation (34) implies the dependence of the shear-strain distribution on the material constants γ_i and n, the geometrical parameter a, and the shear strain γ_a at the periphery of the punch. Because of the simple assumption about shear, the radius a appears as a length scale rather than the plate thickness b, and this is reasonable. Calculation shows that, along with increasing shear strain γ_a at the periphery of the punch, the localization of the shear deformation becomes more severe, although the shear strains increase everywhere in the deformation zone (*see* Fig.5). This is in agreement with observations.

Once the strain γ_a reaches the value γ_i, the critical shear strain for adiabatic shear instability, some new phenomenon is likely. Corresponding to the strain γ_i, the adiabatic shear stress–strain curve reaches its peak stress τ_M. Also, according to the distribution of shear strain obtained previously, the stresses in the region $r>a$ remain less than τ_M, even though $\tau=\tau_M$ at $r=a$. Thus, further deformation will lead to a post-instability shear deformation at $r=a$, in which $\gamma>\gamma_i$ along the descending segment of the adiabatic shear stress–strain curve (*see* Fig.6); the requirement on stress indicates an unloading process at $r>a$. Bearing in mind that the stress state has not exceeded the peak stress, and that no shear instability has occurred there, the unloading is easy to visualize. In the case of finite deformation it is reasonable to suppose that the strain attained is 'frozen in' (*see* Fig.6). Accordingly, a shear-strain discontinuity will appear at the periphery due to the adiabatic shear instability. Clearly, this strain discontinuity will lead to an abrupt shear deformation in a narrow band, and eventually to a crack.

Figure 7 shows how the deformation and stress state of the plate, except for the plug, vary with penetration. It can be seen that before instability both shear strain and stress increase with penetration, but that the post-instability shows a descending stress and a frozen-in strain, i.e. the strain and displacement of the element retain the values attained at the occurrence of the shear instability.



6 Schematic representation of formation of shear-strain discontinuity (the dashed lines represent unloading)



7 Variation of shear stress and strain with penetration at periphery of punch in plate for mild steel

What is still not known is how wide the shear-strain discontinuity is and at what stage it becomes a visible crack and exhausts the plate's resistance to penetration. As for the former, the adiabatic assumption can only give zero width, and it seems that the more accurate thermoplastic shear instability consideration might provide a means of dealing with the problem.[25] The onset of cracking is more difficult to cope with and, to some extent, lies at the core of the problem of ductile fracture. Apparently, all these issues are challenging to current research.

Approximate expressions and general conclusions

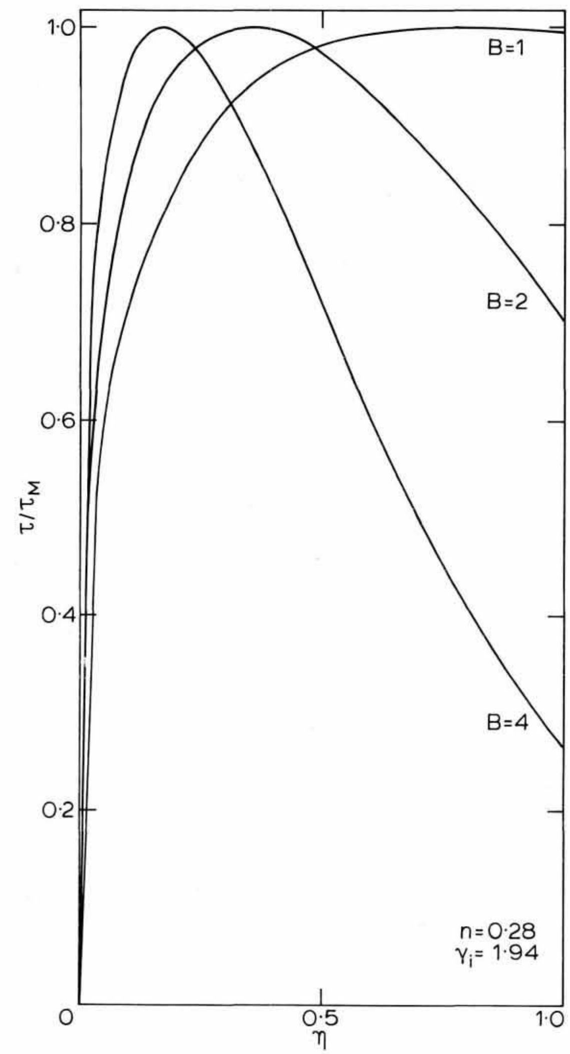
Despite the physical difficulties mentioned in the preceding section, it is possible, based on the approach proposed, to make estimates of the punch load–displacement relations and the energy absorption. In this section two additional assumptions are introduced, one concerning the strain distribution in the plate, and the other the criterion of perforation.

The strain distribution has been obtained and is given by equation (34). Its implications for critical shear strain, the shear-strain discontinuity at the periphery of the punch, and the frozen-in deformation in the plate at post-instability, have been interpreted.

It has also been pointed out that, in this description only, it is impossible to account for the whole plugging process, especially the post-instability stage, without some knowledge of crack initiation. Equation (34) may be approximated to

$$\left(\frac{\gamma}{\gamma_a}\right)=\left(\frac{a}{r}\right)^{1/n}\quad\ldots\ldots(35)$$

The physical implication of equation (35) can be explained as follows. Supposing $\theta\equiv\theta_a$ at $r>a$; then equations (20) and



8 Variation of dimensionless punch load on unit area τ/τ_M fractional penetration η and dimensionless plate thickness

(32) would lead to equation (35). After comparing equations (35) and (34), it is apparent that equation (35) provides the smoothest strain distribution for all shear strains. Also, at post-instability the plate does not stop being deformed, contrary to what was shown in the preceding section, and indeed the deformation continues, but is still localized. Therefore, at a given stress level this strain distribution will allow the plate to absorb most of the plastic-deformation energy.

Substituting equation (35) into equation (33) and then integrating, a simple relation between the penetration and the shear strain at the periphery of the punch can be obtained as

$$\xi=\frac{n}{1+n}\gamma_a \qquad (36)$$

where $\xi=p/a$. Combining equations (36) and (26), the punch load–penetration relationship can be obtained. It can be imagined that the relationship between the forces acting on a unit area of the peripheral surface of the punch and the dimensionless penetration ξ must be very similar to the shear stress–strain curve (*see* Fig.4). All these are one-peak curves with decaying terminations; they are very like the punch load–penetration autographic diagrams obtained for various metals by Chang and Swift[1] (*see* Fig.1). The ascending portion is limited to $\gamma<\gamma_i$. For most metals, γ_i is ~ 1 and $n<0\cdot 5$. In accordance with equation (36), the ascending portion corresponds to the penetration depth $p\approx a$. When the plate is not very thin, it almost always undergoes post-instability deformation.

The influences of the material parameters are now examined. Obviously, the higher the adiabatic shear strength, the higher the peak load the plate can sustain. It should be emphasized that the critical shear strain plays a crucial role in plugging. On the one hand, the larger the critical shear strain, the longer the tail of the load–penetration curve, while on the other hand, the larger the critical shear strain, the greater the delay in the onset of the instability. For a given critical shear strain and penetration, varying the index n changes the load–displacement curve, but not so much as does varying the critical shear strain. Transforming to

$$\eta=\frac{p}{b}=\frac{\xi}{B} \qquad (37)$$

where $B=b/a$ and $\eta=p/b$ is the fractional penetration, the geometrical configuration parameter B is substituted into the load–displacement relationship, thus showing that, the thicker the plate, the earlier the occurrence of the peak load, or the sooner the onset of instability in plugging (*see* Fig.8). For instance, in the case of mild steel plate with $B=2$, the instability begins at a penetration of about one-third of the thickness of the plate.

The more significant aspect in plugging, namely the energy absorption, is now considered. Instead of taking into account the strain discontinuity, crack initiation, coalescence, and frozen-in deformation of the plate at the post-instability stage, it is assumed that the punch process terminates at

$$l=b \qquad (38)$$

Apparently, this assumption is not correct because of the initiation and coalescence of cracks. But considering that, for most metals, the post-instability deformation can still absorb considerable amounts of energy, and because the punch–load displacement relationship obtained by combining equations (36) and (26) resembles the experimental formulae of Chang and Swift,[1] the assumption represents a reasonable simplification. Undoubtedly, this approach will provide an upper limit to the energy absorption in plugging.

Substituting equations (26) and (36) into equation (15) and letting $l=b$, the non-dimensional energy absorption can be expressed as

$$\bar{e}=\frac{e}{2\pi a^2 b\tau_M}$$
$$=\int_0^B\left(\frac{1-n}{n}\frac{\xi}{\gamma_i}\right)^n \exp\left\{\frac{n}{n+1}\left[1-\left(\frac{1-n}{n}\frac{\xi}{\gamma_i}\right)^{n+1}\right]\right\}d\xi \qquad (39)$$

It is easy to see that the energy absorption is a function of the geometrical parameter B and the two material parameters γ_i and n. When $m\ll M$, the energy absorption and the critical impact velocity will be

$$e\approx ab^2 \quad \text{and} \quad v_{0c}\approx\frac{a^{1/2}b}{M^{1/2}} \qquad (40)$$

if the integral (39) is a linear function of B or

$$e\approx a^2 b \quad \text{and} \quad v_{0c}\approx\frac{ab^{1/2}}{M^{1/2}} \qquad (41)$$

if the integral is a constant. An empirical expression such as equation (4) lies between approximations (40) and (41).

Table 1 Material parameters and energy absorptions, for two different plate thicknesses, and three plate materials

Plate material	n	γ_i	τ_M, MNm^{-2}	$\bar{e}$ $B=1$	$B=2$	$\hat{e}$, kJm^{-3}* $B=1$	$B=2$
Mild steel	0·280	1·94	487·7	0·930	1·775	2·85	5·44
Aluminium alloy	0·075	0·438	359·6	0·390	0·414	0·88	0·94
Titanium	0·170	0·326	625·3	0·370	0·375	1·45	1·47

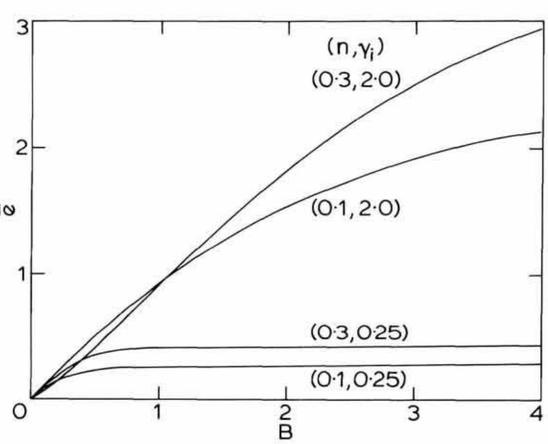
*$\hat{e}=2\pi\tau_M\bar{e}$ is the energy absorption per unit volume.

According to equation (15), the integrated function in equation (39) is, in fact, the shear resistance, so that equation (40) corresponds to constant shear stress, while equation (41) corresponds to a case in which the shear stress drops to zero after a certain penetration. This is why the thickness of the plate appears more influential in equation (40) than in equation (41). Reverting to Figs.4 and 8, it is evident that the theoretically predicted energy absorption must lie in the region between equations (40) and (41). This is in agreement with observations.

Figure 9 shows the dependence of non-dimensional energy absorption on the thickness of plate up to twice the diameter of the punch, for values of critical shear strain of 0·25 and 2, and values of the index n of 0·1 and 0·3, which covers most metals. Roughly speaking, both material parameters benefit from energy absorption. However, it seems that the critical shear strain is more influential than the index on the non-dimensional energy absorption, especially for thick plates. Thick plates made from material of low critical shear strain have exhausted their capacity to absorb energy before perforation; in other words the resistance of the plate to plugging has vanished. This does not mean that increasing the thickness cannot enhance the energy absorption, due to the definition of the non-dimensional energy $\bar{e}$, but it does imply at this stage that the increase of energy absorption is proportional to the plug thickness shown in equation (41), rather than thickness squared, as in equation (40). No doubt this is the least efficient way of absorbing energy.

Generally speaking, nearly all the non-dimensional energy-absorption curves are convex upwards, and therefore the energy-absorption mode of equation (40) is the most efficient. But, as remarked above, this only corresponds to a constant resistance, and thermoplastic shear instability must occur sooner or later, say at $p \leqslant a$, so this situation is unrealizable in real plugging operations.

From the above considerations of energy absorption, it can be concluded that increasing plate thickness is helpful



9 Dimensionless relation of energy absorption $\bar{e}$ and plate thickness B for various indices n and critical shear strains γ_i

only for materials of larger critical shear strain. However, the mode given by equation (40) is the most optimistic albeit an unrealistic one.

It is helpful to examine some examples. In Table 1 the material parameters and energy absorptions for two different plate thicknesses are listed for mild steel, aluminium alloy, and titanium. The material parameters are based on the data in Ref.22. Because it has the largest critical shear strain, the greatest index, and quite a large adiabatic shear strength, steel possesses the greatest capacity for energy absorption. As for titanium, although it has a very high strength, the absorbed energy is very limited as compared to steel. Obviously, this is because of the very low critical shear strain. Titanium is an excellent structural material due to its lightness and high strength, but for the failure mode concerned here, namely plugging resistance, it loses its superiority. In this sense, it can be seen that plugging and related processes represent a novel failure feature.

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References

1. T. M. CHANG and H. W. SWIFT: *J. Inst. Met.*, 1950–51, **78**, 119–146.
2. W. JOHNSON and R. A. C. SLATER: Proc. Int. Conf. 'Manufacturing technology', 1967, CIRP–ASTME, 825.
3. M. E. BACKMAN and W. GOLDSMITH: *Int. J. Eng. Sci.*, 1978, **16**, 1.
4. D. G. SOPWITH: 'Penetration of armour plate', 22; 1947, O. B. Press.
5. A. A. ILYUSHIN and V. S. LENSKY: 'Strength of materials', (English translation, ed. S. C. Redshaw); 1967, Oxford, Pergamon Press.
6. K. E. SPELLS: *Proc. Phys. Soc.*, 1951, **B64**, 212.
7. R. F. RECHT and T. W. IPSON: *J. Appl. Mech. (Trans. ASME, E)*, 1963, **30**, 384.
8. A. PYTEL and N. DAVIDS: *J. Franklin Inst.*, 1963, **276**, 394.
9. H. R. MINICH: No. DA–31–124–ARC(D)–67, 1964.
10. J. AWERBUCH and S. R. BODNER: *Int. J. Solids Struct.*, 1974, **10**, 671.
11. I. MAROM and S. R. BODNER: *Int. J. Mech. Sci.*, 1979, **21**, 685.
12. M. E. BACKMAN and S. A. FINNEGAN: 'Metallurgical effects at high strain rates', (ed. R. W. Rohde *et al.*), 531; 1973, New York, Plenum Press.
13. R. F. RECHT: *J. Appl. Mech.*, 1963, Paper No.63 WA37.
14. R. BALENDRA and F. W. TRAVIS: *Int. J. Mech. Tool Des. Res.*, 1970, **10**, 249.
15. A. I. O. ZAID and F. W. TRAVIS: *Int. J. Mech. Sci.*, 1974, **16**, 373.
16. A. I. O. ZAID and F. W. TRAVIS: 'Mechanical properties at high rates of strain', (ed. J. Harding), 417; 1974, London, Bristol, The Institute of Physics.
17. W. JOHNSON: 'Impact strength of materials'; 1972, London, Edward Arnold.
18. H. C. ROGERS: 'Adiabatic shearing – a review', Drexel University Rep., 1974.
19. H. C. ROGERS: *Ann. Rev. Mater. Sci.*, 1979, **9**, 283.
20. C. J. IRWIN: DREV–R–652/72, Canada, 1972.
21. R. S. CULVER: *Exp. Mech.*, 1972, **12**, 398.
22. R. S. CULVER: 'Metallurgical effects at high strain rates', (ed. R. W. Rohde *et al.)*, 519; 1973, New York, Plenum Press.
23. A. G. ATKINS: *Int. J. Mech. Sci.*, 1980, **22**, 215.
24. G. I. TAYLOR and H. QUINNEY: *Proc. R. Soc.*, 1934, **A413**, 307.
25. Y. L. BAI: Proc. Int. Conf. on 'Metallurgical effects of strain-rate deformation and fabrication', (ed. M. A. Meyers and L. E. Murv), 277; 1980, New York, Plenum Press.