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Tensile and shear instabilities in tensile tests on rods and sheets

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Tensile and shear instabilities in tensile tests on rods and sheets

Results of researchers in the fields of room-temperature and cryogenic tensile testing are reviewed with particular reference to metallographic details. This review leads to the proposition of a mechanism of shear localization in bands in both rod and sheet tensile specimens. This mechanism is thermoplastic shear instability. The concept of 'tensile instability' is not precise. It is shown that tensile instability is a result of a blend of thermoplastic and geometrical softening. This has been confirmed in the case of sheets and thermoplastic shear instability seems to be the more likely cause of the initiation of localized necking. Based on this, some new approaches and explanations of the mechanism of ductile fracture in a rod tensile specimen are proposed. MT/766

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List of symbols

A=area
B=fraction of plastic work per unit volume converted into heat
c_v=specific heat
$d\lambda$=multiplier, *see* equation (11)
k=1 for adiabatic conditions
< 1 for non-adiabatic conditions
K=constant
l=thickness of plastic zone
$m=2n/(n+1)$
M=material constant $\left[=\dfrac{\rho c_v}{\left\{B\left|\left(\dfrac{\partial\tau}{\partial T}\right)_{\gamma,\dot{\gamma}}\right|\right\}}\right]$
n=strain-hardening exponent
P=load
s=thickness
t=time
T=temperature
w=width
W_p=plastic work per unit volume
γ=shear strain
$\dot{\gamma}$=shear strain rate
ϵ=natural strain
$\dot{\epsilon}$=strain rate
κ=thermal diffusivity
ρ=density
σ=normal stress
τ=shear stress
ϕ=angle of localized neck

Subscript
i=instability strains

In the tensile testing of rods, tensile instability occurs at the maximum in the applied load. After this load maximum further plastic deformation is confined to a thinned or necked region of the rod. Within the neck a non-uniform triaxial tension is induced. The triaxial tension is most severe at the axis of the specimen on the plane of minimum cross-section. Bridgman[1] and others have discussed the state of stress within a neck as a function of the width of the neck and the neck radius. With further plastic deformation the neck becomes more pronounced and eventually fracture occurs in the region of the plane of minimum section.

Using the condition of maximum load and taking the area reduction and the rate of strain hardening into account, a simplified theory for the onset of tensile instability has been deduced and referred to by many authors: *see,* for example, Backofen.[2] For a material which exhibits power-law hardening ($\sigma=K\epsilon^n$) the axial strain at which necking begins is equal to the strain-hardening exponent.

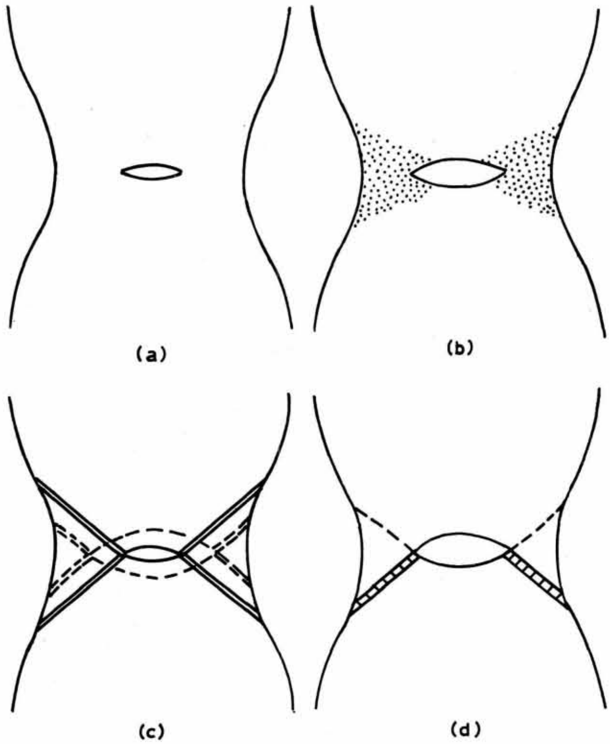
In the same way as for a rod, an annealed sheet becomes unstable in uniaxial tension when the axial strain is equal to the strain-hardening exponent. This is referred to as diffuse necking. With a cold-worked sheet localized necking is observed. In this case at an axial strain equal to twice the strain-hardening exponent the neck forms along a narrow band which is inclined at a characteristic angle to the tensile axis. The angle is governed by the necessity for zero normal strain along the band and for an isotropic material this angle is equal to approximately 54·7°.[3]

From detailed metallographic studies a number of researchers, particularly Rogers,[4] have described the microstructural events which lead to ductile fracture in the neck of rod tensile specimens. Metallographic evidence leaves no doubt that during uniform and unstable tensile deformation voids nucleate and grow in the metal. The voids nucleate at inclusions, grain-boundary triple points, and, in some cases, grain boundaries and within grains. At a critical plastic strain there is clear evidence for void coalescence by localized shear-band formation.

The purpose of this paper is to clarify the concepts concerning tensile instability in rods and sheets and to outline a physically reasonable and theoretically consistent mode of shear-band formation in rods based on the metallographic evidence of previous workers. To do this it is necessary to review briefly the important metallographic work in room-temperature tests as well as outline the research that has been carried out in the field of cryogenic tensile testing.

Rod tensile tests

Two types of ductile fracture that are most often observed in tensile specimens are the cup-and-cone and the double-cup fractures. The latter was apparently first reported by Pumphrey.[5]



a lenticular crack; *b* growth by general deformation; *c* growth by localized shear; *d* cup-and-cone fracture

1 Crack growth in tensile fracture

An exhaustive investigation into tensile tests on OFHC copper has been described by Rogers.[4,6] In specimens which had been heated in hydrogen before testing, at the onset of necking some voids were observed at grain boundaries and grain-boundary triple points. With increasing strain many voids formed along grain boundaries parallel to the tensile axis and larger voids appeared in the minimum section. The concentration of strain was in bands of shear between one void and another at angles between 30° and 40° to the tensile axis. For specimens which had been heated in vacuum or nitrogen before testing, after a large amount of plastic deformation most of the voids formed within the grains. At the centre of the rods the voids were observed to have grown rapidly forming a crack by intense shear deformation. The resulting fracture surfaces showed parabolic dimple patterns indicative of shear deformation. Once again the angle of the bands of intense shear with respect to the tensile axis was about 30°–40°. The final shear was catastrophic leading to a double-cup fracture.

After numerous subsequent investigations, particularly those of Broek,[7] Cox and Low,[8] and Hahn and Rosenfield,[9] it is accepted that in commercial metals and alloys voids will nucleate heterogeneously at inclusions and/or second-phase particles.[10] The mechanism will either involve particle–matrix decohesion or particle cracking, depending on the mechanical properties of the particle in relation to those of the matrix.[7–11] The importance of the volume fraction of inclusions and second-phase particles on ductility has been described by Edelson and Baldwin.[12]

Chin *et al.*[13] have given a simple explanation for the formation of cup-and-cone and double-cup fractures. Referring to Fig.1*a* it is assumed that a lenticular crack has formed as a result of void coalescence. Further growth of the crack can take place by general deformation as shown in Fig.1*b* or localized shear deformation as shown in Fig.1*c*. Localized shear is favoured by reduced strain-hardening capacity and plane-strain conditions and therefore it is expected to occur later in the process. In a pure material growth is expected to occur like this resulting in a double-cup fracture. In an impure or commercial material, growth occurs by localized shear and new voids are generated. As deformation continues voids are nucleated along these bands. The final failure is along just such a weakened band.

Zener[14] and Petch[15] have suggested that final separation in a tensile testpiece occurs by adiabatic shear leading to the characteristic conical shape of a cup-and-cone fracture. Puttick[16] discounted this suggestion and preferred to state that final separation occurred by 'tensile failure'. Further, he was able to suppress the formation of the conical surfaces by using a 'stiffer' testing machine. In the same way Chin *et al.*[17] decreased the size of the final shear by using a stiffer machine. The importance of the stiffness of the machine–specimen system, first described by Orowan,[18] in relation to the resulting fracture must be explained satisfactorily in any proposed mechanism of fracture in tensile tests.

From all the metallographic evidence, the sequence of events which leads to ductile fracture in a tensile test can be summarized as follows[19]:

(i) voids form after a critical amount of strain
(ii) at the maximum in the applied load necking begins and with further straining void growth occurs
(iii) at a critical volume fraction of voids or at a critical mean free path between the voids, strain concentrates in narrow bands connecting the voids
(iv) separation occurs along these bands.

Cryogenic tensile testing

At cryogenic temperatures tensile deformation of a number of metals and alloys results in serrated stress–strain curves which are similar to those observed at higher temperatures due to dynamic strain aging.[20] At testing temperatures below 45 K serrated flow has been observed in stainless steel.[21] This behaviour has been attributed to the formation of strain-induced martensite. However, serrated flow has been observed in pure aluminium[13,22,23] and other metals which do not undergo martensitic transformations. In cryogenic tests it has been observed that the frequency of discontinuities increases with decreasing temperature until a limiting temperature is reached at which flow is totally discontinuous.

At testing temperatures as low as that of liquid helium the specific heat of metals is small and it is proportional to the cube of the temperature. If there is a local increase in plastic strain, the low specific heat allows the heat generated by the deformation to heat the plastic zone and localize deformation further. Therefore the load decreases suddenly until the deformation ceases and the specimen is quenched. Basinski[22] has pointed out that localized heating can produce an appreciable softening effect and that the heat produced could be localized at least for the time required for plastic flow to occur and therefore the deformation would become unstable. Basinski has demonstrated this both experimentally and analytically.

It has been observed by Rogers[24] that all the cryogenic deformation studies emphasize that adiabatic shear analyses define the conditions under which a strain perturbation will be exaggerated. However, the heat generated does not cause localization, an outside disturbance is required to nucleate the instability as has been observed experimentally by Basinski[22] and Chin *et al.*[13]

Chin *et al.*[13] observed that as the testing temperature was decreased there was a transition from cup-and-cone to shear fractures. The shear fractures obtained at very low temperatures (<50 K) indicate catastrophic shear has occurred on a small number of planes after a certain degree of necking. This shear deformation will occur under nearly adiabatic conditions and the width of the shear bands will be very small.

For a given material, specimen geometry, and strain rate, as the testing temperature is decreased, the temperature generated as a result of the plastic work becomes more important until it becomes the dominant factor in the deformation and, under these conditions, the possibility of unstable plastic flow becomes correspondingly higher. The activation of shear instabilities then becomes the main factor responsible for unstable plastic flow.

It is also possible to obtain markedly different types of fracture surface as a function of applied strain rate. Dodd *et al.*[25] tested 304L stainless steel rod tensile specimens at strain rates of between 10^{-3} and $10^3\,s^{-1}$. At the quasistatic rates conventional cup-and-cone fractures were obtained. However, at elevated strain rates ($>10^2\,s^{-1}$) shear fractures were observed. In the shear fractures, shear deformation was confined to a small number of planes indicating that adiabatic shear was the mechanism for final fracture.

Adiabatic and thermoplastic deformation

An adiabatic process is defined as one in which heat is not lost to the surroundings. Thus, adiabatic plastic deformation is defined as deformation during which the heat generated as a result of the plastic work is confined to the plastic zone. Practically, in any process of plastic deformation a proportion of the heat generated will be conducted away from the plastic zone. However, this proportion is minimized in high-strain-rate deformation such as occurs in ordnance firings, explosive and high-rate forming, and high-speed machining.

A number of analyses of adiabatic deformation has been made in relation to cryogenic testing,[13,22] high-speed machining,[26] and torsion tests.[27] In all of these analyses the importance of thermal softening in relation to strain and strain-rate hardening is discussed. When the rate of thermal softening exceeds the combined hardening effects of the strain and strain rate, the result is unstable flow which often leads to catastrophic failure.

It is an oversimplification to label plastic deformation processes in which heat transfer cannot be ignored as adiabatic processes. In these cases the term thermoplastic deformation is suggested.

TENSILE INSTABILITY

For a metal that exhibits power-law hardening, necking (or tensile instability) begins when the axial strain is equal to the strain-hardening exponent, neglecting thermal and rate effects. Thermal effects can be taken into account by taking the flow stress to be a function of strain, strain rate, and temperature, neglecting history effects:

$$\sigma = f(\epsilon, \dot{\epsilon}, T) \quad \ldots\ldots (1)$$

Therefore an increment of stress, $d\sigma$, is given by

$$d\sigma = \left(\frac{\partial\sigma}{\partial\epsilon}\right)_{\dot{\epsilon},T} d\epsilon + \left(\frac{\partial\sigma}{\partial\dot{\epsilon}}\right)_{\epsilon,T} d\dot{\epsilon} + \left(\frac{\partial\sigma}{\partial T}\right)_{\epsilon,\dot{\epsilon}} dT \quad \ldots\ldots (2)$$

Necking begins at maximum load,

$$dP = 0 \quad \ldots\ldots (3)$$

where $dP = A\,d\sigma + \sigma dA$. Substituting equation (2) into equation (3) and remembering that $d\epsilon = -dA/A$, at the onset of necking the following relation is obtained

$$\left(\frac{\partial\sigma}{\partial\epsilon}\right)_{\dot{\epsilon},T} + \left(\frac{\partial\sigma}{\partial\dot{\epsilon}}\right)_{\epsilon,T}\frac{d\dot{\epsilon}}{d\epsilon} + \left(\frac{\partial\sigma}{\partial T}\right)_{\epsilon,\dot{\epsilon}}\frac{dT}{d\epsilon} - \sigma = 0 \quad \ldots\ldots (4)$$

For simplicity the differential term $dT/d\epsilon$ may be expressed as $Bk\sigma/\rho c_v$, where B is the fraction of the plastic work converted into heat, $k=1$ for adiabatic conditions and $k<1$ for non-adiabatic conditions, ρ is the density, and c_v the specific heat. k can be shown to depend on the ratio $l/\sqrt{\kappa t}$ where l is the thickness of the plastic zone, κ the thermal diffusivity, and t the time of flow. Taking the strain rate to be constant throughout the test, and rearranging, necking will begin when

$$\left(\frac{\partial\sigma}{\partial\epsilon}\right)_{\dot{\epsilon},T} = \sigma\left\{\frac{Bk}{\rho c_v}\left|\left(\frac{\partial\sigma}{\partial T}\right)_{\epsilon,\dot{\epsilon}}\right| + 1\right\} \quad \ldots\ldots (5)$$

The first term on the right-hand side illustrates the effect of thermal softening and the second term, unity, is the geometrical-softening effect.

If the isothermal rate-constant strain hardening is given by the power law, $\sigma = K\epsilon^n$, then the axial strain corresponding to the onset of necking is given by,

$$\epsilon_i = \frac{n}{\left\{\frac{Bk}{\rho c_v}\left|\left(\frac{\partial\sigma}{\partial T}\right)_{\epsilon,\dot{\epsilon}}\right| + 1\right\}} \quad \ldots\ldots (6)$$

If the plastic deformation is carried out very slowly, then $k \rightarrow$ zero and $\epsilon_i \rightarrow n$. However, for many practical circumstances of tensile testing $k \neq 0$ and the thermal-softening term cannot be neglected. Under non-isothermal conditions the axial strain at the onset of tensile instability will be smaller the smaller the values of ρ and c_v and the larger the value of the rate of thermal softening, $(\partial\sigma/\partial T)_{\epsilon,\dot{\epsilon}}$. The thermal softening effect illustrates how it is possible for tensile instability to initiate at axial strains which are less than the strain-hardening exponent.

Equations (5) and (6) are equally applicable to the onset of necking in rods and diffuse necking in sheets. For simplicity in the derivation of equation (6) the stiffness of the machine has been assumed to be infinite. However, machine-stiffness effects are of importance in cryogenic tensile testing[14] and this can be incorporated, in principle, in the above derivation.

It is clear from the above description that the so-called 'tensile instability' is a phenomenon which is dependent on the geometrical configuration of the specimen as well as material effects. From this aspect tensile instability differs from 'shear instability' as it is shown in the next section that the criterion for the onset of shear instability contains only material parameters.

SHEAR INSTABILITY

In torsion testing at constant shear strain rate, using a simple analysis similar to the one above, Culver[28] derived the following equation for the onset of an adiabatic shear instability ($k=1$):

$$\frac{B\tau\left|\left(\frac{\partial\tau}{\partial T}\right)_{\gamma,\dot{\gamma}}\right|}{\rho\, c_v\left(\frac{\partial\tau}{\partial\gamma}\right)_{\dot{\gamma},T}} = 1 \quad \ldots\ldots (7)$$

If the isothermal rate-constant strain hardening is assumed to be of the power-law type then

$$\gamma_i=\frac{n\rho c_v}{B\left|\left(\frac{\partial\tau}{\partial T}\right)_{\gamma,\dot{\gamma}}\right|} \quad \ldots\ldots (8)$$

where γ_i is the instability strain. It is interesting to note that the only difference between equation (8) and equation (6) is the absence of a geometrical-softening term.

It has been shown analytically by Bai[29] that the criterion given in equation (7) remains the same even when heat transfer cannot be neglected, i.e. for non-adiabatic conditions. Therefore for non-adiabatic conditions equation (7) is the criterion for the onset of thermoplastic shear instability.

It has been observed that catastrophic adiabatic shear failures are extremely localized.[24] When a shear instability is thermoplastic, theoretically the width of the shear band broadens. In the limiting case of isothermal deformation, the shear-band width is infinite, and thermoplastic instability is suppressed. In general, though, when a metal is plastically deformed and heat is therefore generated, the strain, strain rate, and mechanical and physical properties determine the quantity of heat and the manner in which it is conducted into the bulk of the material.

In metal-forming processes, plastic deformation is inhomogeneous and this in itself may lead eventually to thermoplastic shear instability. Rogers[24] has observed that in compressive bulk-forming operations where there is high interfacial friction between the dies and the workpiece, high temperatures can be generated locally even in quasistatic processes because of inhomogeneous deformation. It has been shown by Jenner *et al.*[30,31] that this concept of thermoplastic shear instability can be used successfully to describe the onset of surface cracking in upset forging. The use of the thermoplastic shear instability concept is a logical extension of the observations of Rogers and others.

There are two conditions which must be fulfilled for the activation of a thermoplastic shear instability in an element subjected to a combined stress state:

(i) the ratio $\tau\Big/\left(\frac{\partial\tau}{\partial\gamma}\right)_{\dot{\gamma},T}$ must be equal to $\frac{\rho c_v}{B\left|\left(\frac{\partial\tau}{\partial T}\right)_{\gamma,\dot{\gamma}}\right|}$ which may be regarded as a characteristic material constant

(ii) the combined stress state must be one that allows pure shear deformation. There are two orthogonal pure shear planes in plane strain which coincide with the maximum shear-stress planes. Also, there are two non-orthogonal planes of pure shear in plane stress.

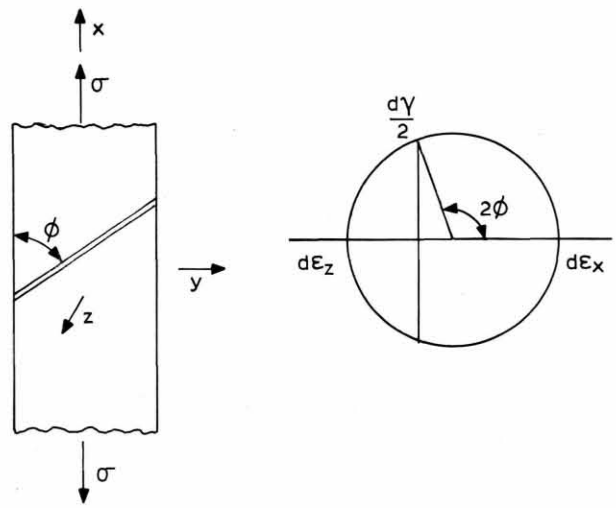
Necking of sheet specimens

TENSILE INSTABILITY ANALYSIS OF NECKING OF SHEET SPECIMENS

It has been shown that diffuse necking in sheets occurs when the axial strain is given by

$$\epsilon_i=\frac{n}{\left\{\frac{Bk}{\rho c_v}\left|\left(\frac{\partial\sigma}{\partial T}\right)_{\epsilon,\dot{\epsilon}}\right|+1\right\}} \quad \ldots\ldots (6)$$

for a material which exhibits power-law hardening. Excluding machine-elasticity effects, this equation indicates that the axial strain for the onset of diffuse necking can be lower than the strain-hardening exponent if thermal effects cannot be neglected. This would be so, for example, at an elevated strain rate (assumed constant in the derivation of equation (7)) and for metals which have a relatively low density and atomic weight because of the constant value of the atomic heat capacity 25·1 J mol^{-1} excluding low-temperature cases.



2 *a* local neck in sheet tensile specimen and *b* graphical determination of local necking angle

It is straightforward to derive the corresponding condition for localized necking. Referring to Fig.2 since the area $A=ws$, then the incremental change in area $dA=w\,ds+s\,dw$. However, because of constancy of volume and the assumption that the width at localized necking remains constant,

$$d\epsilon=2ds/s \quad \ldots\ldots (9)$$

Substituting this result into equation (4) and rearranging leads to

$$\epsilon_i=\frac{n}{\left\{\frac{Bk}{\rho c_v}\left|\left(\frac{\partial\sigma}{\partial T}\right)_{\epsilon,\dot{\epsilon}}\right|+\frac{1}{2}\right\}} \quad \ldots\ldots (10)$$

Once again if thermal effects can be neglected this equation reduces to $\epsilon_i=2n$. In cases where thermal effects cannot be ignored in, for example, elevated strain-rate tests, this strain for the onset of localized necking is necessarily less than $2n$.

APPLICATION OF THERMOPLASTIC SHEAR INSTABILITY CRITERION TO LOCALIZED NECKING IN SHEETS

There is some inconsistency in discussing localized necking in terms of tensile instability. This is because although tensile instability is used to derive a condition for the onset of localized necking, the orientation of the band is taken to coincide with the pure shear direction. To unify the explanation of all the aspects of localized necking in sheets the concept of shear instability is used.

To apply the criterion given by equation (7) to localized necking, the following assumptions are made:

(i) the isothermal rate-constant stress–strain relation is given by $\sigma=K\epsilon^n$
(ii) the von Mises yield function equals the plastic potential
(iii) geometrical softening is ignored.

The first two assumptions allow the following expression for the multiplier, $d\lambda$, in the Levy–Mises equations to be obtained.

$$d\lambda = \frac{2K}{W_p{}^m} dW_p \quad \ldots\ldots (11)$$

where $m=2n/(n+1)$ and W_p is the plastic work per unit volume.

The plastic shear strain increment is then given by

$$d\gamma = \frac{2K}{W_p{}^m} dW_p\tau \quad \ldots\ldots (12)$$

The shear stress on the pure shear plane is given by

$$\tau = \frac{\sigma}{2}\sin 2\phi \quad \ldots\ldots (13)$$

where the angle of the localized neck ϕ is determined by the conditions of constancy of volume and plane stress, Fig.2. Similarly, the shear strain increment is given by

$$\frac{d\gamma}{2} = \frac{3}{4} d\epsilon \sin 2\phi \quad \ldots\ldots (14)$$

The plastic-work increment per unit volume is then

$$dW_p = \frac{4\tau d\gamma}{3\sin^2 2\phi} \quad \ldots\ldots (15)$$

From equations (11) and (15) it is possible to derive the following expression for the ratio $\tau \Big/ \left(\frac{\partial\tau}{\partial\gamma}\right)_{\dot\gamma, T}$,

$$\frac{\tau}{\left(\frac{\partial\tau}{\partial\gamma}\right)_{\dot\gamma, T}} = \frac{\sqrt{6K}}{m}\left(\sin 2\phi\right) W_p{}^{1-m/2} \quad \ldots\ldots (16)$$

Now for shear instability to initiate this ratio must be equal to the characteristic material constant

$$M = \frac{\rho c_v}{\left\{B\left|\left(\frac{\partial\tau}{\partial T}\right)_{\gamma,\dot\gamma}\right|\right\}}$$

The critical shear strain for the onset of shear instability is

$$\gamma_i = \frac{\sqrt{6K}}{2-m}\left(\sin 2\phi\right) W_p{}^{1-m/2} \quad \ldots\ldots (17)$$

or

$$\gamma_i = \frac{m}{2-m} M = nM \quad \ldots\ldots (18)$$

and since

$$\epsilon_i = \frac{2\gamma_i}{3\sin 2\phi}$$

then:

$$\epsilon_i = \frac{\gamma_i}{\sqrt{2}} = \frac{n}{\sqrt{2}} M \quad \ldots\ldots (19)$$

In order to obtain some estimates of the axial strains for localized necking, data due to Culver[28] are used, Table 1. Columns 5 and 6 show the critical strains required for localized necking caused by thermoplastic shear instability and geometrical softening, respectively. It is clear that for α-Ti and Ti–6Al–4V thermoplastic shear instability is the dominant mechanism in localized necking, whereas Armco iron and 4130 steel are not sensitive to thermoplastic shear instability, and geometrical softening is dominant.

The above results imply that localized necking should result from a combination of thermoplastic shear instability and geometrical softening. Because of the fixed direction of the localized shear band, thermoplastic shear may initiate localized necking, although it is not the dominant factor for those materials which are insensitive to thermoplastic shear instability.

Shear instabilities and fracture in tension tests on rods

The case of necking in a rod tensile specimen is more complex than necking in a sheet. In the latter the stress state is plane stress and remains so and the pure shear band remains in its characteristic orientation during the course of the test. In rods, however, the stress state within the neck is non-uniform. Despite this, however, in relation to the discussion in the above section, thermoplastic shear instability may still be involved to some greater or lesser extent in the initiation of the neck. This is likely because voids, shear bands, and cracks occur in the centre of the rod before they do nearer the surface. Further details of the activation of thermoplastic shear instabilities in the neck of a rod tensile specimen are now given.

In the neck the activation of a thermoplastic shear instability requires that:

(i) the ratio $\tau/(\partial\tau/\partial\gamma)_{\dot\gamma,T}$ reach the characteristic material constant, and

(ii) that there are two voids (or cracks) between which plastic deformation can occur by pure shear. After a critical amount of plastic straining, the activation of the first instability should be in the region of the most severely stressed part of the neck. This is on the axis of the specimen at the plane of minimum section in the neck. The voids between which this instability is initiated can either be pre-existing flaws, or be formed by plastic deformation at inclusions, second-phase particles, grain-boundary triple points, or grain boundaries depending on the nature and cleanliness of the test material. It is possible that the activation of this instability will be suppressed when, even though the ratio $\tau/(\partial\tau/\partial\gamma)_{\dot\gamma,T}$ has reached its characteristic value, there are not a pair of voids with a suitable orientation for pure shear bands to develop. In this case plastic deformation will continue until either the voids move to appropriate positions due to the straining or the stress state changes or a combination of both.

Once a thermoplastic shear instability has been initiated the plastic deformation leading to void coalescence by the void sheet mechanism will occur rapidly. The crack will have an orientation to the tensile axis corresponding to pure shear deformation. When the crack has formed the local stress and strain fields will be considerably more complex, the stress field at the tip of the crack will be important in determining the orientation of the following localized shear bands.

After the first instability has been activated and has formed the central crack, then it is possible to activate further thermoplastic shear instabilities between the tip of the crack and voids adjacent to it. Once again, for the activation of further instabilities the two conditions must be met: $\tau/(\partial\tau/\partial\gamma)_{\dot\gamma,T}$ must reach the characteristic material constant and the orientation of the void with respect to the tip of the central crack must be such as to allow pure shear deformation. It is then possible to see that there will be repeated activation of shear instabilities as plastic straining continues and the central crack will expand radially outwards in a succession of rapid shears.

It has been observed[16] that the length of the final shear failure leading to the characteristic cup shape is dependent on machine stiffness, for constant applied strain rate and specimen geometry. The 'harder' the machine the smaller the cone. Also it has been suggested by Zener[14] and Petch[15]

Table 1 Physical parameter data

Material	Density ρ, kg m^{-3}	Specific heat c_v, 10^{-3} J kg^{-1} K^{-1}	Strain-hardening exponent n	Rate of thermal softening ($-\partial\sigma/\partial T$), 10^9 MN m^{-2} K^{-1}	ϵ_i (equation (19))	$\epsilon_i = 2n$
Al 6061–T6	2707	0·9629	0·075	0·4961	0·309	0·15
Armco iron	7849	0·5024	0·21	0·6821	0·953	0·42
4130 steel	7849	0·5024	0·35	1·2403	0·874	0·7
α-Ti	4533	0·5443	0·17	1·4263	0·23	0·34
Ti–6Al–4V	4421	0·5024	0·08	1·2403	0·112	0·16

*For other materials the rate of thermal softening ($-\partial\sigma/\partial T$) can be taken from standard reference books or from papers such as Ref. 32.

that this final failure is adiabatic. From Orowan's description of unstable ductile fracture[18] it is clear that the interaction of the machine–specimen stiffness with the mechanical and physical properties of the specimen determine the size of the cone. Under some circumstances with soft machines this final failure may be the result of near adiabatic shear. But, on the other hand, with hard machines this deformation certainly does not take place under adiabatic conditions. Both of these extremes of behaviour can be accommodated by using the concept of thermoplastic shear instability.

Near-adiabatic conditions would result in a thin catastrophic shear band whereas non-adiabatic conditions, in which nevertheless local thermal softening is of importance, would result in broader shear bands. Thus, with moderately hard machines the so-called 'final shear' may be much more complex, resulting from the activation of a number of thermoplastic shear instabilities.

Winchell[33] carried out some tensile tests on an alloy steel with the composition Fe–0·48C–20Ni. These tests were carried out at a strain rate of about 10^{-4} s^{-1} and an environmental temperature of 0°C. The material was quenched and tempered. After fracturing test pieces were sectioned and etched. The bands of intense shear consisted of austenite. For the tempered martensite to transform to austenite the temperature could not have been less than 200°C and could have been higher. Winchell's experimental evidence confirms that the local temperature rise along the intense shear bands is large even though the tests were carried out at a low rate and at 0°C. For other materials tested at similar rates and environmental temperatures the temperature rise may well be in excess of 200°C along the intense shear bands depending on the physical and mechanical properties of the test material.

In summary, the sequence of events which leads to ductile fracture of a rod tensile specimen is postulated to be as follows:

1. Uniform macroscopic plastic deformation with the formation of a small number of voids at inclusions, second-phase particles, or grain boundaries.
2. As the plastic deformation develops and the heat generated by the plastic work diffuses into the specimen, a neck develops at some critical plastic strain and at the maximum load. Depending on the material and the testing conditions the onset of necking may be caused by shear instability, geometrical softening, or both. After further straining, a number of intense shear bands form which join pairs of voids. The shear bands are the result of the activation of thermoplastic shear instabilities. These bands are expected to occur initially in the region of the specimen axis on the plane of minimum section.
3. With further plastic straining the central crack grows radially by the shear instability mechanism.
4. Final separation leading to the characteristic conical cup shape is by the activation of either one or multiple shear instabilities. The number of instabilities and the length of the shear bands are dependent on the material properties as well as the machine stiffness.

In the development of the theoretical framework for the thermoplastic shear instability, the effects of hydrostatic pressure are ignored. Since concern here has been with tensile tests carried out with no superimposed hydrostatic pressure, this is not an unreasonable working hypothesis. A future refinement of the theory will allow for the effects of superimposed hydrostatic pressure on ductility.

Centre bursting or chevron cracking in wire or rod which has been drawn or extruded is similar in many ways to the formation of cup-and-cone fractures. It has been shown[34] that this type of fracture occurs in the region of the axis of the wire when the hydrostatic tension is large. Because of the number of important similarities between centre bursting and fracture in rod tensile specimens, it is suggested that the mechanism resulting in centre bursting is the activation of thermoplastic shear instabilites.

Conclusions

The onset of necking in rods and sheets as well as localized necking in sheets have been discussed taking into account thermal softening. Thermoplastic shear instability has been proposed as the mechanism of shear localization between voids in the neck of a rod tensile specimen. For sheets the factors contributing to the initiation of localized necking are thermal and geometrical softening. For α-titanium and a titanium alloy it has been shown that thermoplastic shear instability is the predominant factor causing localized necking whereas for Armco iron and 4130 steel the predominant factor is geometrical softening. Materials in which thermoplastic instability will be the predominant feature in localizing necking will have a low density and a relatively low atomic weight.

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